

NONSINUSOIDAL ELECTROMAGNETIC WAVES

Electromagnetic waves in communications are traditionally thought of to have sinusoidal time variation. Strictly speaking, this is only true for unmodulated carriers that transmit information at the rate zero, but the concept is good enough for engineering purposes as long as the effective bandwidth of the modulated carrier is small compared with the carrier frequency. There are applications where one does not want to be restricted by this condition. Methods using very wide frequency bands have been developed under the name spread spectrum techniques. The use of nonsinusoidal carriers may be viewed as an extension of wide band signaling to bandwidths of many gigahertz. The concepts of Fourier analysis can be used for a qualitative understanding of these techniques, but they are usually not applicable to a quantitative analysis.

The introduction of nonsinusoidal waves has created a great deal of confusion and frustration for electrical engineers, particularly for those working in the EMC and EMI fields. This situation is a result of the historical emphasis in the radar and communications fields upon theoretical descriptions, hardware realizations, and educational curricula based on sinusoidal functions. A good way to break free from this constraint is to consider the use of nonsinusoidal electromagnetic waves as an extension of digital signal processing and digital circuits to radiated waves. Gates, counters and shift registers use enormous frequency bandwidths, but this has not prevented their use. Coils and capacitors are not much good for digital circuits, but we have learned to build circuits with other components such as switches and diodes. The best applications of digital circuits were not in traditional fields of the sinusoidal theory, such as telephony, but in new fields, such as computers. These principles apply to nonsinusoidal waves as well. The typical hardware known from sinusoidal waves, such as LC filters, wave guides and cavity resonators, is useless, but we know how to build filters, resonators, radiators and generators based on other components. The best applications of nonsinusoidal waves are *not* in competition with existing uses of sinusoidal waves but are uses never mentioned for sinusoidal waves. Let us summarize these statements in three points:

- Electromagnetic waves do not have to vary with time like sinusoidal functions but can vary like more general functions.
- Equipment for nonsinusoidal waves is different from that for sinusoidal waves, but we can build it.
- The typical applications of nonsinusoidal waves are different from the typical applications of sinusoidal waves.

The next problem is to find useful nonsinusoidal waves. There are two approaches. We can use waves that are best from the standpoint of our technology, and we can use waves that are best from the standpoint of a desired application. Let us first turn to waves that are best from the standpoint of technology. Semiconductor technology works best if used with voltages or currents that are either turned on or off. This means that two-valued functions of time are favored. Figure 1 shows a system of two-valued functions. These are the so-called Walsh functions. There are many other systems of two-valued functions. The great advantage of the Walsh functions is that they have a widely accepted notation. We can write $\text{sal}(k, t/T)$ and $\text{cal}(k, t/T)$ just as we can write $\sin 2\pi kt/T$ and $\cos 2\pi kt/T$. The fraction $k/T = f$ is called frequency for the sinusoidal functions and sequency for the Walsh functions. The sequency of a Walsh function is one half the number of its jumps or zero crossings in Fig. 1, as one may readily verify for $k=1, 2, \dots, 8$. Many frequency meters count the number of zero crossings of an applied sinusoidal voltage, and they can thus be used to measure the sequency of an applied Walsh voltage.

Let us feed a current $i(t) = I \text{cal}(3, t/T)$ with the time variation of a Walsh function to a Hertzian electric dipole, which is a very short dipole. The electric and magnetic field strengths produced by this current are shown in Fig. 2. The electric field strength has three components:

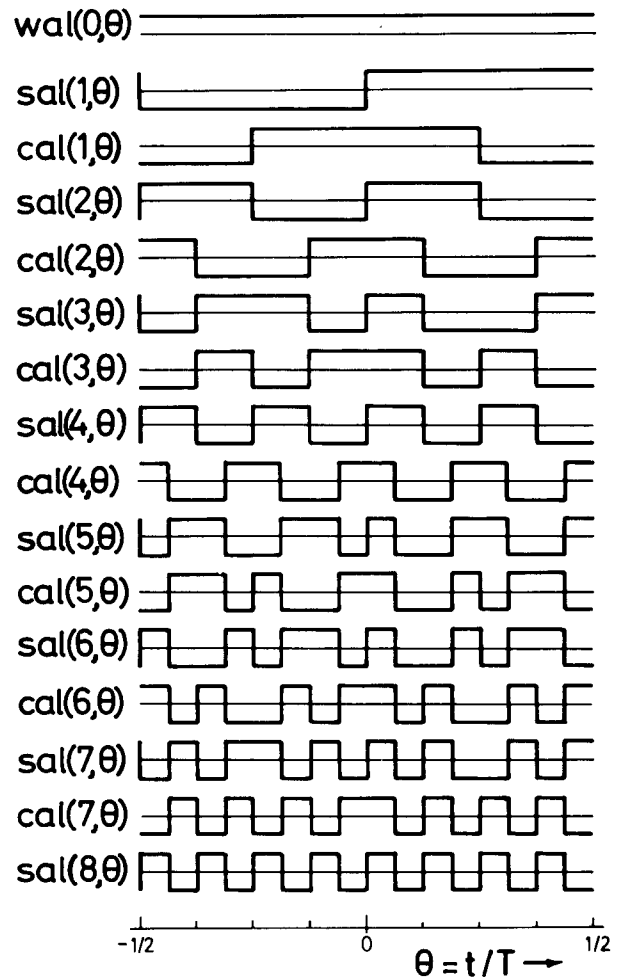


Fig. 1. The first sixteen Walsh functions. $\text{wal}(2k, \theta) = \text{cal}(k, \theta)$, $\text{wal}(2k-1, \theta) = \text{sal}(k, \theta)$; $\theta = t/T$.

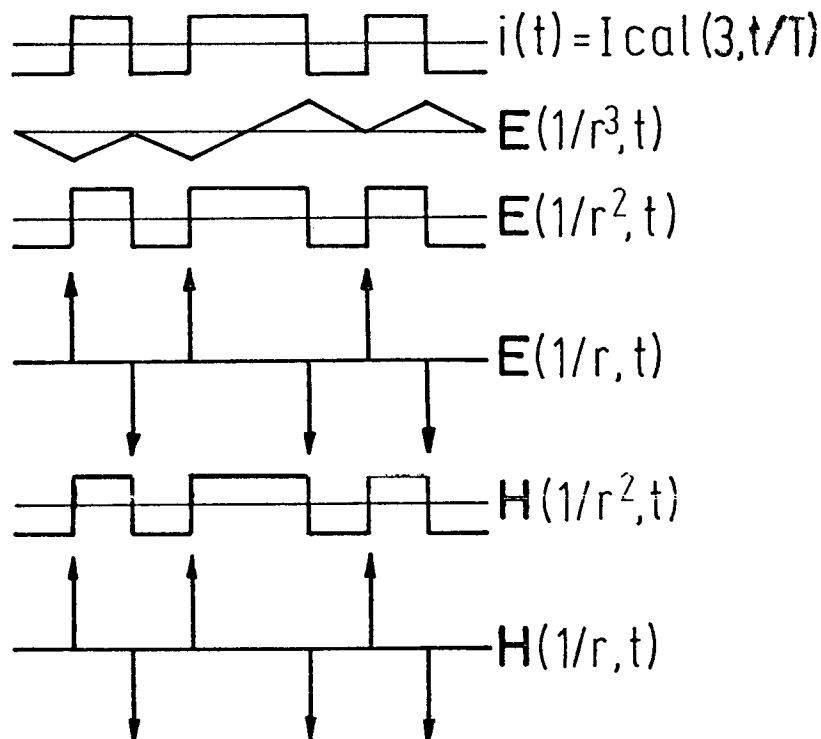


Fig. 2. Walsh shaped current $i(t)$ flowing in a Hertzian electric dipole and the electric and magnetic field strengths produced in the induction zone ($1/r^3$), the near field ($1/r^2$) and the far field ($1/r$).

1. The induction component $E(1/r^3, t)$ decreasing with distance like $1/r^3$; its time variation is that of the integral of the antenna current $i(t)$.
2. The near field component $E(1/r^2, t)$ decreasing with distance like $1/r^2$; its time variation is that of the antenna current $i(t)$.
3. The far field component $E(1/r, t)$ decreasing with distance like $1/r$; its time variation is that of the first derivative of the antenna current $i(t)$.

Let us note that a sinusoidal current $i(t) = I \cos 2\pi(3t/T) = I \cos 2\pi ft$ produces the same three components with the time variation $\int \cos 2\pi ft \, dt = (2\pi f)^{-1} \sin 2\pi ft$, $\cos 2\pi ft$ and $(d/dt) \cos 2\pi ft = -2\pi f \sin 2\pi ft$, but these three components differ by a phase shift only and do not have the radically different time variation of the components of the field strength produced by the Walsh current in Fig. 2.

The components of the magnetic field strength produced by the Hertzian electric dipole vary in the near and far zone like the components of the electric field strength according to Fig. 2; there is no component of the magnetic field strength that decreases like $1/r^3$.

A practical current cannot be switched infinitely fast as the current $i(t)$ in Fig. 2. The finite switching times cause the Dirac pulses of $E(1/r, t)$ and $H(1/r, t)$ to become short pulses with a large but finite amplitude.

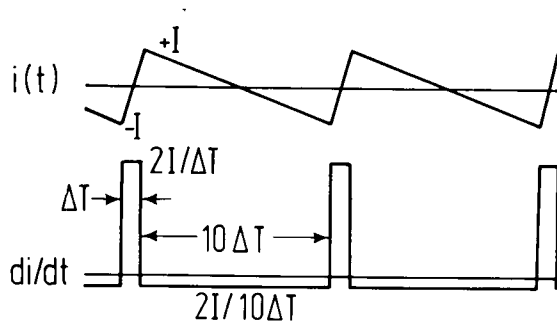


Fig. 3. Sawtooth antenna current $i(t)$ used to discriminate an airplane with conducting aluminum surface from rain and ground clutter.

The short pulses of the electric and magnetic field strength in the far field imply that their power is spread over a very wide frequency band. One might think that this causes interference in many radio channels. This is not so. The spreading of the radiated power over an extremely wide frequency band implies that the power density is extremely low. A receiver for sinusoidal carriers cannot distinguish this nonsinusoidal carrier from the natural background noise. Vice versa, a receiver for Walsh waves will not be disturbed by sinusoidal waves, since the power of the sinusoidal waves is spread over a very wide *sequency* band, and the sinusoidal waves now disappear in the natural background noise. This disappearance of signals in noise is well known and understood from spread spectrum communications. The applications are the same. The main difference is that Walsh waves spread the power over a band from about 1 MHz to 10 GHz, which cannot be done practically with a wideband signal modulated onto a sinusoidal carrier.

Let many statistically independent nonsinusoidal carriers with very wide frequency band be radiated simultaneously. These carriers can be separated from each other if they are produced by antenna currents having the time variation of either the functions $\text{sal}(k, t/T)$ or the functions $\text{cal}(k, t/T)$ in Fig. 1; this is in complete analogy to using either the carriers $\sin 2\pi kt/T$ or $\cos 2\pi kt/T$ but not sinusoidal and cosinusoidal carriers with the same frequency. Each nonsinusoidal carrier will produce a little crosstalk to a certain frequency channel. The crosstalk from any carrier is statistically independent from that of the others. Hence, the crosstalk from many nonsinusoidal carriers to a certain frequency channel is summed statistically. To maintain the signal-to-noise ratio in the presence of n nonsinusoidal carriers one must increase the signal power in the frequency channel in first approximation by \sqrt{n} , but one can now operate in essence twice as many independent radio channels in the same very large frequency or sequency band. This is by no means in contradiction to general concepts of communications. We have known for a long time that the transmittable information increases proportionate to the bandwidth and the logarithm of the signal power. What is new is that we know now how to provide more independent channels with fixed information rate for each channel, rather than a fixed number of channels with a higher information rate for each channel.

The increase of the number of independent radio channels by the use of nonsinusoidal carriers is still a long range project with many educational, administrative and technical hurdles in its way. The practical importance of more radio channels needs no discussion. The simultaneous use of sinusoidal, Walsh and perhaps other nonsinusoidal carriers calls for EMC standards that will be far more complex than those for our present allocation of radio channels based on sinusoidal carriers only.

Let us turn to an example of a wave that is best from the standpoint of a desired application. Figure 3 shows a sawtooth current $i(t)$. If this current is fed into a Hertzian electric dipole one obtains an electric field strength in the far zone that varies like the shown first derivative di/dt . Let this electric field strength be produced by a radar transmitter and let the wave hit an airplane. The highly conductive aluminum surface causes a polarity reversal of the electric field strength of the returned wave. Raindrops or the dry surface of the earth are insulators; they do not reverse the polarity of the electric field strength of the returned wave (They reverse the polarity of the magnetic field strength). Nothing more than a fast diode is needed to discriminate between the wave returned by the airplane and a wave returned by raindrops or the ground. In addition, one may still use the known moving target indicator techniques for improved discrimination.

The polarity of the electric field strength of a sinusoidal wave is of course also reversed by a conducting scatterer or reflector, but a reversal of the polarity cannot be distinguished from a delay by half a period, and the effect can thus not be utilized.

More information on applications and the implementation of equipment may be found in the book "Applications of Walsh Functions and Sequency Theory", edited by H. Schreiber and F. Sandy, and published by IEEE, New York 1974 (\$ 10 for IEEE members, \$ 13 for nonmembers). This book also contains a list of hundreds of references.

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