# AN EXPLANATION OF BROADBAND INTERFERENCE

The title terminology "broadband interference" will be changed to "broadband response", since the word "interference" denotes something undesirable. In this article the broadband response is the central theme, and is considered beneficial.

The best way to discuss broadband response is to reflect on the definition of narrowband response. Consider the circuit below in Figure 1.

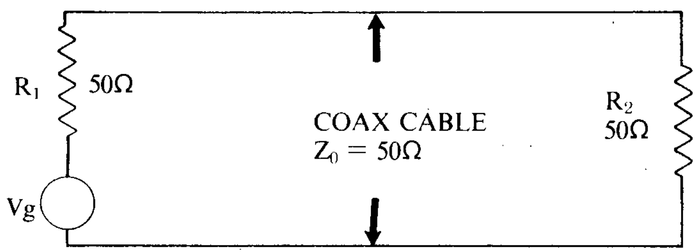



**Figure 1.**

If Vg is an ideal sine wave generator with a frequency of 1 MHz, the response of a spectrum analyzer or an EMI meter connected at $R_2$ would look like that shown in Figure 2.

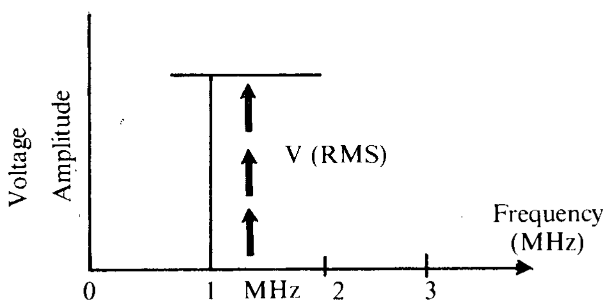



**Figure 2.**

However, if Vg is a distorted sine wave, harmonics would appear at 2 MHz, 3 MHz, 4 MHz, ad infinitum in Figure 2. *This phenomenon suggests that if a source produces all of its energy at one single frequency, it has a narrowband response. If it splits its energy into two or more frequencies, its response is broadband.*

Another source of broadband energy less familiar than the distorted sine wave is a repetitive, rectangular pulse. If this pulse replaces Vg in Figure 1, the spectrum analyzer display would appear as shown in Figure 3.

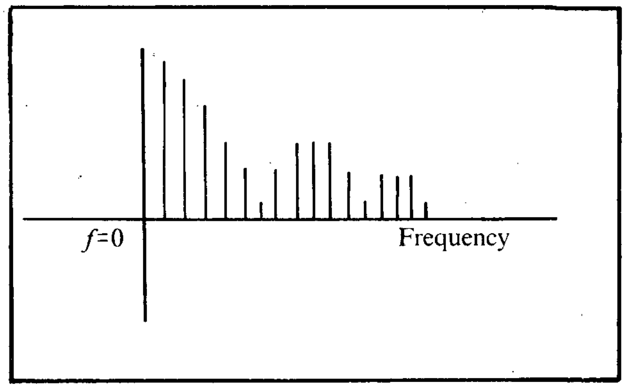



**Figure 3.**

Math models for a cosine wave and a pulse are presented below. A Fourier transform analysis is used because it accurately describes the frequency domain response of a spectrum analyzer to time domain signals.

**FOURIER ANALYSIS OF A COSINE WAVE AND A PULSE**

Equations 1, 2, 3 and 4 are crucial for predicting the frequency domain response of even so simple a waveform as a cosinusoid.

The Fourier transform F(f), of V(t) is as follows.

$$F(f) = \int_{-\infty}^{\infty} V(t)\, e^{-j2\pi ft}\, dt. \qquad \text{Equation 1}$$

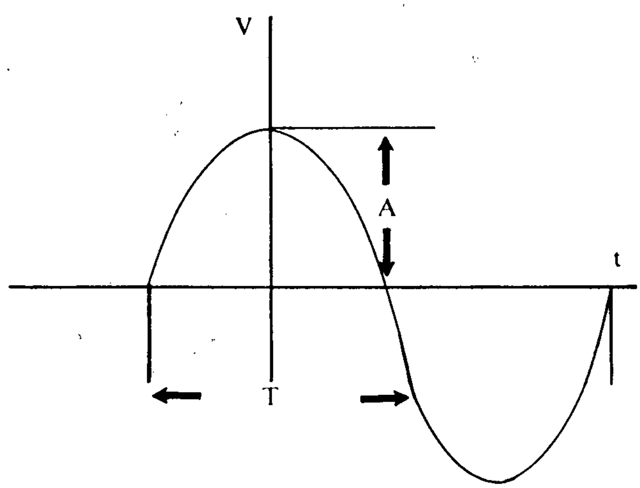



**Figure 4.**

If V(t) is a perfect cosine, (see Figure 4) in the time domain, F(f) in the frequency domain is shown in Equation 2.

$$F(f) = \frac{A}{2}[\delta(f-\tfrac{1}{T})+\delta(f+\tfrac{1}{T})] \qquad \text{Equation 2}$$

$\delta(f)$ is the symbol introduced by Dirac to stand for an impulse that is infinitely brief and infinitely strong. $\delta(f) = 0$ everywhere along the axis except at $f = 0$,

and $\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$

At $f = 0$, $\delta(f)$ can be graphed as a spike of unit height as shown in Figure 5.

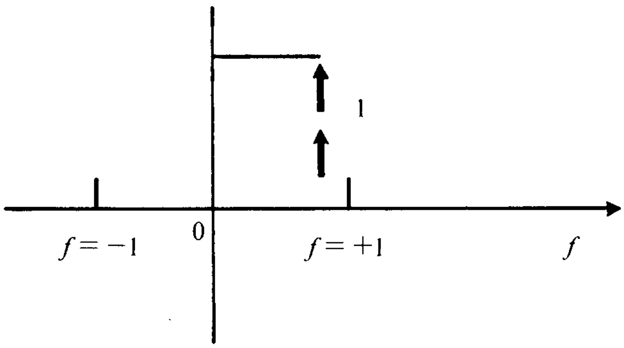



**Figure 5.**

With the above understanding of $\delta(f)$, Equation 2 can be graphed as shown in Figure 6.

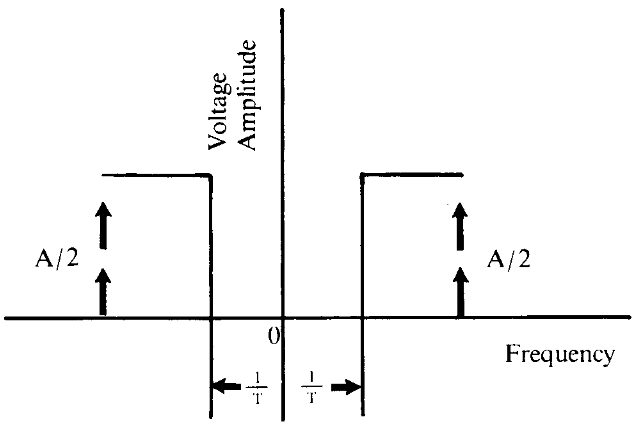



**Figure 6.**

At $(f-\frac{1}{T}) = 0$, $f = \frac{1}{T}$, and at $(f+\frac{1}{T}) = 0$, $f = -\frac{1}{T}$. An analyzer does not display the negative frequency part of the Fourier spectrum; what it *does* display is twice the absolute value of the Fourier transform's positive spectrum:

viz : $2/F(f)/$.

Hence, a spectrum analyzer would display $2|\frac{A}{2}| = A$ and it would paint a line "A" high at a frequency of $\frac{1}{T}$. One further point: an EMI meter will always display the rms value of the signal(s) present within its bandwidth. Therefore, it would indicate .7071A and not the peak value "A", even though it detects the peak of the wave(s) within its impulse bandwidth. See Figure 7.

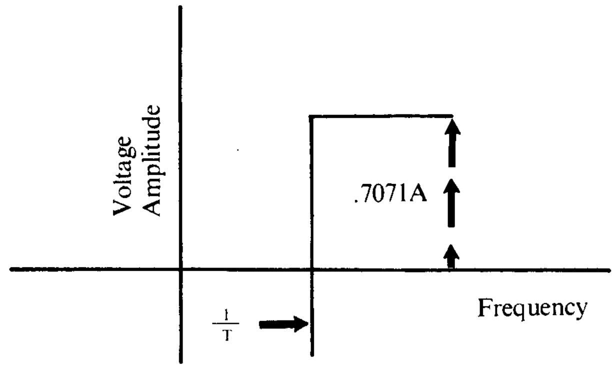



**Figure 7.**

The solution of a Fourier analysis for a rectangular pulse train follows. Figure 8 shows this pulse train as it appears in the time domain.

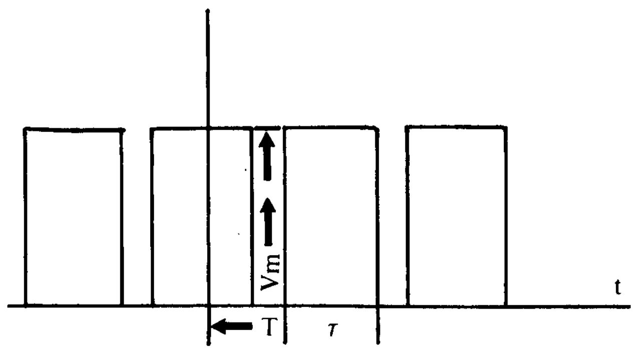



**Figure 8.**

Its Fourier transform is as shown in Equation 3.

$$F(f) = \frac{Vm}{\pi} \sum_{nf-\infty}^{\infty} [\delta\,(f-\tfrac{n}{T})] \frac{\sin \pi\, \tau\, \frac{n}{T}}{n} \qquad \text{Equation 3}$$

Again, the term in brackets $[\delta(f-\frac{n}{T})]$ must be interpreted [because it contains $\delta(f)$] to make sense out of Equation 3. It serves only to reduce to zero all responses in $(f)$ *except* those at $n = \pm1, \pm2, \pm3, \ldots\ldots \pm\infty$. For numerical evaluation purposes it can be set equal to unity.

$$\int_{-\infty}^{\infty} \delta(f-\tfrac{n}{T})\, df \equiv 1 \quad \text{for all } f = \tfrac{n}{T}$$

where $n = 0, \pm1, \pm2, \pm3, \pm4, \ldots \pm\infty$

A graph of Equation 3 is shown below in Figure 9. If the pulse train's period (T) is short enough, an analyzer can resolve the individual frequency components. It would display only the right-hand half of Figure 9 — it would also show each spectral line as being twice the amplitude in Figure 8 and invert all negative lobes. (See Figure 3.)

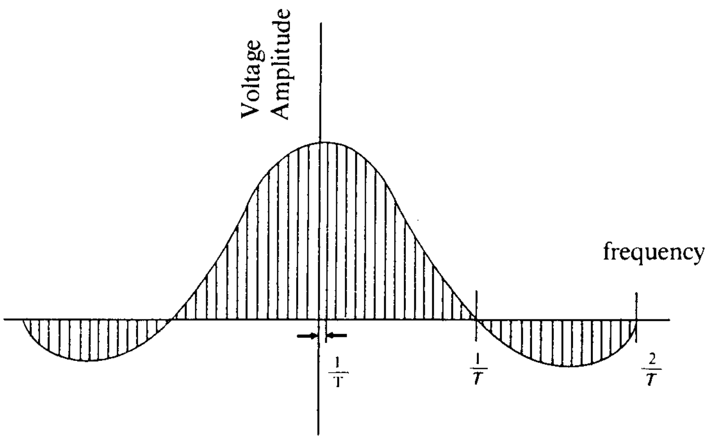



**Figure 9.**

## RESPONSE OF AN EMI METER TO AN UNRESOLVABLE SIGNAL

When an analyzer *cannot* resolve the individual frequency components, a different type of measurement must be made; within the EMI discipline this measurement is called "broadband."

The broadband response of an EMI meter to the pulse train of Figure 8 is explained as follows. All EMI meters are essentially superheterodyne receivers with a peak detector connected to the output of their I.F. strip. Even though frequency translations occur within them, their broadband response can be easily understood if they are pictured as simply an ideal bandpass filter with its input connected to the pulse signal and its output connected to a peak detector. See Figure 10.

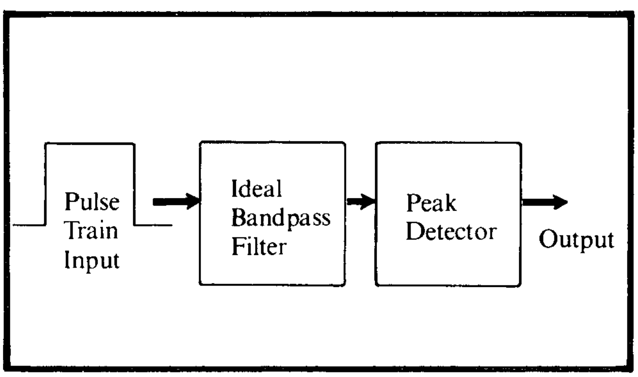



**Figure 10.**

This bandpass filter acts to allow only those frequencies present within its bandpass to get through to the peak detector and rejects all frequencies both below and above its bandpass. Its center frequency is also variable, thus the filter can be pictured as sweeping back and forth along the frequency axis. See Figure 11.

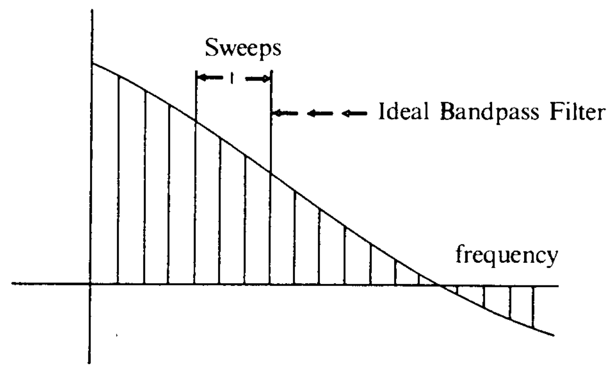



**Figure 11.**

Here, to get the response in the time domain at the peak detector, the *inverse* Fourier transform is used. It must be remembered that Equation 3 is the *frequency domain* solution to the rectangular pulse. Equation 4, given below, is the *time domain* solution to the same pulse.

$$V(t) = \frac{V_m}{\pi} \sum_{n=-\infty}^{\infty} \frac{\sin \pi \tau (\frac{n}{T})}{n} \text{Cos } [2\pi(\tfrac{n}{T})\, t] \qquad \text{Equation 4}$$

Equation 4 states that the mathematically awkward pulse train of Figure 8 is identically equal to the sum of a series of harmonically related cosine waves. The bandpass filter in Figures 10 and 11 sees the infinite series of cosine waves at its input and outputs only in those frequencies lying within its bandpass; i.e., it restricts the limits on $\Sigma$ in Equation 4 to the upper and lower cutoff frequencies of the filter. The peak detector, in turn, detects the maximum value of the sum of the cosine waves in the bandpass and the analyzer displays this peak value at the center frequency of the bandpass filter. *All EMI meters that are used to make MIL-STD-461 measurements do this.*

Restricting Equation 4 to the positive frequency domain, as was done for Equation 3, results in the bandpass response as being equal to twice the absolute value of the inverse Fourier transform; $2/V(t)/$. The maximum value of Equation 4 occurs at $t = 0$, and Equation 4 reduces to evaluating the terms of the summation.

Upper cut-off frequency

$$\frac{2V_m}{\pi} \sum \frac{\sin \pi \tau (\frac{n}{T})}{n}$$

Lower cut-off frequency

That expression, of course, would normally entail the use of a computer since it's fair to say that no one would want to spend his weekends evaluating it with his hand-held calculator. Fortunately, Equation 4 can be approximated very closely by taking the average of the values at the filter's upper and lower cut-off frequencies and multiplying the number of cosine waves simultaneously present within the bandpass. For example,

**Filter Parameters**

Upper cut-off frequency $= 575{,}000$ Hz

$n = (575{,}000)(10^{-3}) = 575$
$n = fT$

Lower Cut-off Frequency $= 425{,}000$ Hz
$n = (425{,}000)(10^{-3}) = 425$

Filter Bandwidth $= 150{,}000$ Hz

**Pulse Parameters**

$T = 10^{-3}$s; $\tau = 10^{-6}$s; $Vm = 1$ volt

$$2/Vt/_{L.C.F} = \frac{2Vm}{\pi}\,\frac{\sin \pi\tau(\frac{n}{T})}{n} = \frac{2(1)}{\pi}\,\frac{\sin \pi(10^{-6})(\frac{425}{10^{-3}})}{425}$$

$$= 1.456 \times 10^{-3}\ V$$

$$2/Vt/_{U.C.F} = \frac{2(1)}{\pi}\,\frac{\sin \pi\,(10^{-6})(\frac{575}{10^{-3}})}{575} = 1.077 \times 10^{-3}\ V$$

Avg $= 1.2665 \times 10^{-3}$V. The number of spectral lines in the filter bandwidth is equal to $n = fT = (150 \times 10^3)(10^{-3}) = 150$. Therefore, $(1.2665 \times 10^{-3}) \times 150 = .190$ V/150 KHz.

Equation 5

Considering that Equation 5 was computed over an Equation 5 filter bandwidth of 150 KHz it is customary to assign units of V/Hz to it.

Normalizing to 1 MHz (MIL-STD-461, for example, uses 1 MHz as a baseline):

$$(.190) \times \left[\frac{1 \times 10^6 Hz}{150 \times 10^3 Hz}\right] = 1.2667\ V/MHz.$$

Converting to dB$\mu$V/MHz;

$$dB\mu v/MHz = 20\ \text{Log}\left(\frac{1.2667V}{10^{-6}V}\right) = 122.05$$

Subtracting 3 dB to account for RMS values yields 119.05 dB$\mu$V/MHz. *A laboratory measurement with an EMI meter will verify this.* This measurement can be performed by using Figure 1. Vg and R1 should be replaced in that Figure with a 50 ohm pulse generator. R2 should be the 50 ohm input impedance of the EMI meter. The pulse amplitude, width, rise/fall times and PRF should be set with an oscilloscope. The meter input should be padded with about 40 dB or the 1 volt pulse may overload it.

Notes: The author wishes to express his gratitude to his collegue Alec Bargman, without whose help this paper could not have been written. Derivations for Equation 2, Equation 3 and Equation 4 are available from the author.

## GENERAL REFERENCES:

1. "The Fourier Transform and Its Applications", Ron Bracewell, McGraw-Hill Book Company, New York.

2. "Reference Data for Radio Engineers", Chapter 42.

3. MIL-STD-461 "Electromagnetic Interference Characteristics Requirements for Equipment."

*This article was written for* ITEM 85 *by Richard Curtis, Astronautics Corporation of America, Milwaukee, WI.*

## APPENDIX 1

The derivations for Equation 2, Equation 3 and Equation 4.

Derivation for the Frequency Domain response of a Cosine wave - Equation 2.

1. $V(t) = A\ \text{Cos}\ 2\pi\,(\frac{1}{T})t = A\ \text{Cos}\ 2\pi\frac{w}{2\pi}t = A\ \text{Cos}\ wt$

where: $w = 2n\,(\frac{1}{T})$

2. $F(f) = \int_{-\infty}^{\infty} V(t)\, e^{-j2\pi ft}dt$

$= A\int_{-\infty}^{\infty} \text{Cos}\ wt\ e^{-j2\pi ft}dt$

since $\text{Cos}\ wt = \frac{e^{jwt} + e^{-jwt}}{2}$,

$$F(f) = \frac{A}{2}\int_{-\infty}^{\infty} e^{jwt}\, e^{-j2\pi ft}dt + \frac{A}{2}\int_{-\infty}^{\infty} e^{-jwt}\, e^{-j2\pi ft}dt$$

$$= \frac{A}{2}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f-\frac{w}{2\pi})t}\, dt + \frac{A}{2}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f+\frac{w}{2\pi})t}\, dt$$

Since The Fourier transform of $1 = \delta(x)$,

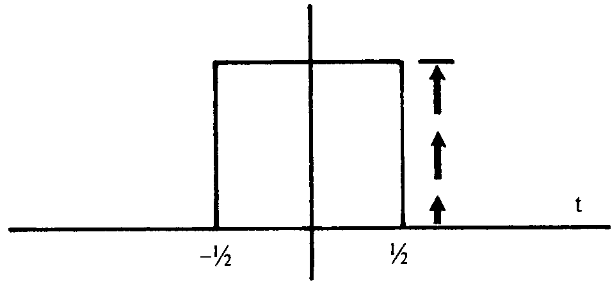
$$F(f) = \frac{A}{2}[\delta(f - {}^{w}/_{2\pi}) + \delta(f + {}^{w}/_{2\pi})]$$

$$= {}^{A}/_{2}\,[\delta(f - \tfrac{1}{T}) + \delta(f + \tfrac{1}{T})] \quad \text{Equation 2}$$

## Derivation for the Frequency Domain response of a positive, repetitive, rectangular pulse - Equation 3.

$$1.\ \Pi(t) = \begin{cases} 0 & /t/ > \frac{1}{2} \\ (\frac{1}{2} & /t/ = \frac{1}{2}) \\ 1 & /t/ < \frac{1}{2} \end{cases}$$

$$\text{AND, III}(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$$




2. When III(t) enters into convolution with a time function, it replicates it. Therefore, the positive repetitive rectangular pulse can be expressed as:

$$V(t) = \frac{1}{T}\,\text{III}\,(\tfrac{t}{T}) * V_m \Pi\,(\tfrac{t}{\tau})$$

Please see figure 7 for the definitions of $V_m$, T and $\tau$.

3. Since convolution of two functions means multiplication of their transforms, the Fourier transform of V(t) above becomes:

$$F(f) = \frac{1}{T}[T\ \text{III}\ (Tf)] \cdot V_m\, \tau\, \frac{\sin \pi \tau f}{\pi \tau f}$$

$$= \frac{V_m}{\pi} \sum_{n=-\infty}^{\infty} \frac{\sin \pi \tau (\frac{n}{T})}{n}\ \delta\,(f - \tfrac{n}{T}) \quad \text{Equation 3.}$$

## Derivation of the Time Domain response for the positive, repetitive, rectangular pulse—Equation 4

1. F(f), for the pulse train, is an even function. Therefore, the inverse Fourier transform for it can be expressed as:

$$V(t) = \int_{-\infty}^{\infty} \frac{V_m}{\pi} \sum_{n-\infty}^{\infty} \frac{\sin \pi \tau (\frac{n}{T})}{n} \operatorname{Cos} 2\pi(\tfrac{n}{T})t\ \delta(f - \tfrac{w}{T})\, df$$

$$= \frac{V_m}{\pi} \sum_{n-\infty}^{\infty} \frac{\sin \pi \tau (\frac{n}{T})}{n} \operatorname{Cos} 2\pi(\tfrac{n}{T})t \int_{-\infty}^{\infty} \delta(f - \tfrac{w}{T})\, df$$

Since $d(f - \frac{w}{T}) = df$, and $\int_{-\infty}^{\infty} \delta(f)\, df \equiv 1$,

Then;

$$V(t) = \frac{V_m}{\pi} \sum_{-\infty}^{\infty} \frac{\sin \pi \tau (\frac{n}{T})}{n} \operatorname{Cos} 2\pi\, (\tfrac{n}{T})t \quad \text{Equation 4.}$$