How to design 3rd-order cauer bandpass filters

A 3rd-order Cauer BPF can be designed to replace the less selective Chebyshev BPF.

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A 1998 ITEM filter article explained how to design and assemble a 3rd-order, 3-resonator Chebyshev audio bandpass filter (BPF). The BPF was suitable to satisfy the requirements of the European Norm 55020 test specification as listed in Appendix B2, p. 35 of the specification.[1] Although the selectivity of the BPF was relatively broad, the 30-dB bandwidth was nevertheless adequate to meet the requirements of the test specification. However, there are many other filtering applications where a more selective response is needed and where the simplicity of design and assembly is within the capabilities of the average EMI test technician or engineer. This article will discuss the design and assembly of such a BPF.

BACKGROUND

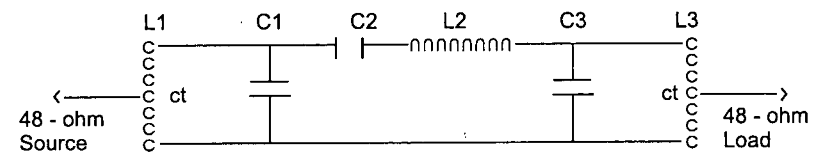
A 3rd-order Chebyshev BPF design was used in an *ITEM 98* article because it was the simplest design that satisfied the test specification for which it was intended. The Chebyshev BPF response is characterized by equi-ripple maximum passband attenuation of preferably less than 0.1 dB, a corresponding equi-level minimum passband return loss of preferably more than 16.4 dB and upper and lower stopband attenuation skirts that constantly increase at a slope of about 21 dB per octave.

This filter type is named after Pafnuty Lvovitch Chebyshev (1821-1894), a famous Russian mathematician and academician. While touring Europe in 1852 to inspect various types of machinery, windmills, water turbines, railways, etc., Chebyshev became interested in the mechanical linkage used in Watt's steam engine to convert the reciprocating motion of the piston rod into rotational motion of a flywheel that was needed to run factory machinery. Chebyshev noted that Watt's piston had zero lateral discrepancy at three points in its cycle, and concluded that a somewhat different linkage would lead to a discrepancy of half of Watt's and would be zero at five points in the piston cycle. Chebyshev then wrote a paper, now considered a mathematical classic, that laid the foundation for the topic of best approximation of functions by means of polynomials. It is these same polynomials that were originally developed to improve the reciprocating to rotational linkage in a steam engine that now find application in the design of the Chebyshev passive LC filters![2]

The schematic diagram, attenuation response and design parameters of the Chebyshev BPF from the 1998 *ITEM* article are shown in Figures 1 and 2 to allow later comparison with a more selective BPF to be discussed.

Advantages of the 3rd-order Chebyshev

- The BPF is easy to assemble because it requires only three resonators.



Component Values	
C1,3	= 0.33 µF
C2	= 1.12 µF
L1,3	= 53.3 mH, center-tapped
L2	= 15.7 mH

Design source and load impedance = 192 ohms

Design Parameters & Frequencies (Hz)		
F1,2,3= F_C	=	1.20 k
B3dB	=	3.30 k
B30dB	=	9.306 k
$F3dB_{LO}$	=	390
$F3dB_{HI}$	=	3.690 k
$F30dB_{LO}$	=	152
$F30dB_{HI}$	=	9.458 k
B30/B3 Shape Factor	=	2.82
3-dB %BW = 100*3.3/1.2	=	275%
Min. Passband Return Loss	=	20 dB

Figure 1. 3rd-order Chebyshev bandpass filter designed for a center frequency of 1.20 kHz and a 3-dB bandwidth of 3.30 kHz. L1 and L3 are center tapped so the bandpass filter can be terminated with source and load impedances of 48 ohms. See Figure 2 for the associated attenuation response curve.

- All resonators are tuned to the same frequency which is the center frequency of the BPF.
- Because of the relatively poor skirt selectivity, the Q requirements for the inductors and the component tolerances are less stringent than the more selective filter types.
- The component values and frequencies associated with the Chebyshev BPF are easily calculated using tables of published normalized data available in filter design handbooks.[3, 4, 5]

Disadvantages of the 3rd-order Chebyshev

- The poor skirt selectivity of the 3rd-order Chebyshev BPF may be inadequate for applications requiring a more abrupt rise in the upper and lower attenuation skirts
- If a BPF application needs only a certain minimum level of stopband attenuation (for example 30 dB), any additional attenuation above this minimum level is wasted.

This second disadvantage is the case of the Chebyshev stopband response that continues to rise higher and higher although the higher levels serve no useful purpose. This disadvantage of the Chebyshev has been eliminated in a more complex filter type known as the "Cauer," which can be designed to have a specific level of minimum stopband attenuation. The additional increase in attenuation available in the Chebyshev is exchanged in the Cauer for improved selectivity.

INTRODUCING THE CAUER

The Cauer filter response (a more selective response than the Chebyshev) is named after the German mathematician and network theorist, Prof. Dr. Wilhelm A.E. Cauer (1900–1945), who did much of the work developing this filter type. This filter response is also known by the generic term of "elliptic" because its design is based on elliptic integrals.

The Cauer BPF is a more complex filter type than the Chebyshev and its stopband attenuation response is distinguished by the presence of zeroes in the transmission response and minimum levels of attenuation that may be specified by the designer. In the case of the 3rd-order Cauer BPF, there are two transmission zeroes, one below and one above the passband. The Cauer passband is similar to that of the Chebyshev in that it has equi-ripple maximum passband attenuation of preferably less than 0.1 dB and a corresponding equi-level minimum passband return loss of preferably more than 16.4 dB.

The schematic diagram of the 3rd-order Cauer BPF is similar to that of the 3rd-order Chebyshev BPF shown in Figure 1, and the only schematic difference is that the series resonator of the Chebyshev BPF is replaced with two parallel-resonant circuits connected in series between the input and output shunt resonators. Figure 3 shows the Cauer BPF and Figure 4 shows its attenuation and return loss responses. In Figures 3 and 4, the designations of F2 and F3 indicate the frequencies of the two transmission zeroes. The level of the minimum 30-dB stopband attenuation is indicated by the designation A_S.

The series branch, comprised of resonators 2 and 3 in Figure 3, is usually connected between the tops

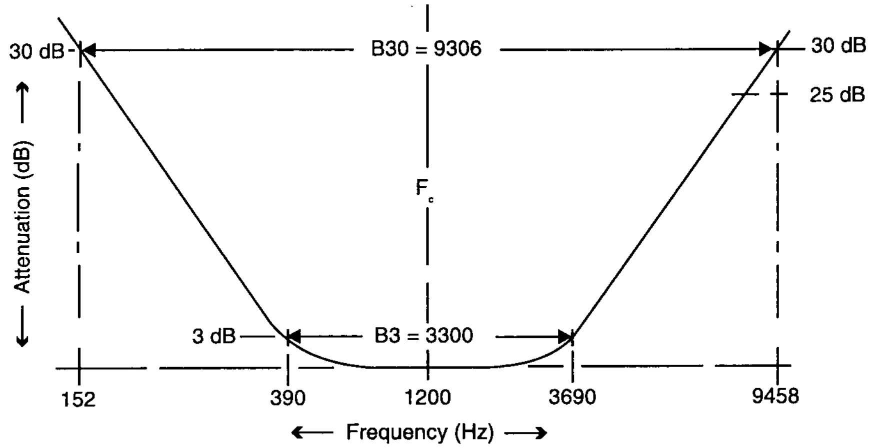
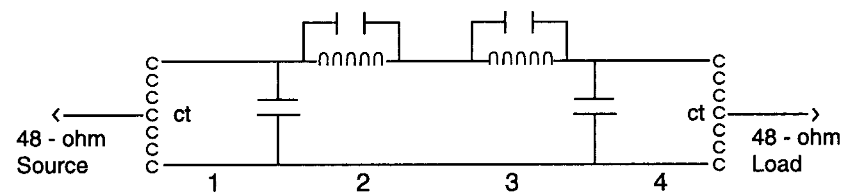


Figure 2. The expected attenuation response of the 3rd-order Chebyshev bandpass filter shown in Figure 1.



Component Values	Design Parameters & Frequencies (Hz)	
C1,4 = 0.2824 μF	$F1,4 = F_C$ = 1.20 k	$F30dB_{LO}$ = 217
C2 = 0.03625 μF	F2 = 7.522 k	$F30dB_{HI}$ = 6.617 k
C3 = 1.424 μF	F3 = 191.4	B3dB = 3.30 k
L1,4 = 62.294 mH, center-tapped	$F3dB_{LO}$ = 391	B30dB = 6.400 k
L2 = 12.349 mH	$F3dB_{HI}$ = 3.692 k	A_S = 30 dB
L3 = 485.21 mH	B30/B3 Shape Factor = 1.94	
Design source and load	3-dB %BW = 100(3.3/1.2) = 275%	
impedance = 192 ohms	Min. Passband Return Loss = 20.486 dB	

NOTE:
The Cauer passband minimum return loss, center frequency, 3-dB bandwidth and the high and low 3-dB frequencies are made to be approximately the same as the Chebyshev BPF to demonstrate that the Cauer BPF provides a more selective stopband response than the Chebyshev for the same passband parameters. A Cauer minimum stopband attenuation level, A_S, of 30 dB was specified to agree with the Chebyshev stopband level of 30 dB at 152 Hz and 9.458 kHz.

Figure 3. 3rd-order Cauer bandpass filter designed for a center frequency of 1.20 kHz, a 3-dB bandwidth of 3.30 kHz and a passband minimum return loss of about 20 dB. L1 and L4 are center tapped so the bandpass filter can be terminated with source and load impedances of 48 ohms. See Figure 4 for the insertion and return loss response curves.

shown that these lower inductor values of resonators 2 and 3 are more convenient to realize than the original inductor values, and for this reason the series branch will be connected between the center taps on L1 and L4.

The ratio of the largest capacitor value (or inductance value) to the smallest value is, in this case, C3/C2 = 5.696/0.145 = 39.3. This is a relatively wide spread in component values and is a consequence of the relatively wide percentage bandwidth that was required for this particular design. The 3-dB percentage bandwidth is calculated by the equation 100 $(B3/F_C)$ where B3 is the 3-dB bandwidth and F_C is the center frequency of the bandpass filter. For the BPF shown in Figure 3, the percentage bandwidth is 100 (3.3k/1.2k) = 275 percent. For a 3-dB relative bandwidth of 100 percent, the component-value spread is much smaller and the design is easier to realize. For example, for a 3rd-order Cauer design with similar parameters as of resonators 1 and 4 as shown in the schematic diagram. However, if the reactances of the capacitors and inductors in resonators 2 and 3 are too high, the series branch can be connected to taps on inductors L1 and L4 to lower the branch reactances to a more convenient level. For example, if L1 and L4 are center tapped (as shown in the diagram), then by connecting the series branch to these center taps the series-branch inductances become one quarter of their original values and the capacitances become four times greater than their original values.

The original design values of L2 and L3 were 12.349 mH and 485.21 mH and the values of C2 and C3 were 0.03625 μF and 1.424 μF. If this series branch is connected between the center taps of L1 and L4, the inductances of L2 and L3 will become 1/4 of their original values, or 3.087 mH and 121.3 mH. The capacitances of C2 and C3 will become four times greater than their original values, or 0.145 μF and 5.696 μF. It will be

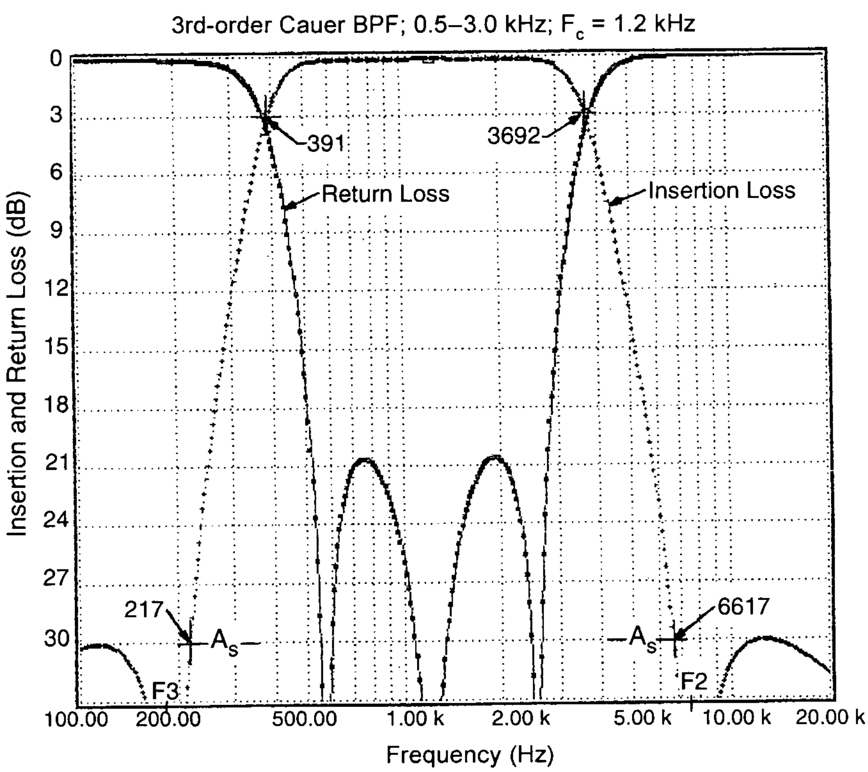


Figure 4. The curves show the computer-calculated insertion loss and return loss responses of the 3rd-order Cauer BPF. The 3-dB and 30-dB frequencies are marked on the insertion loss response.

shown in Figure 3, and a 3-dB bandwidth of 1.2 kHz and a center frequency of 1.2 kHz, the inductor and capacitor value spread is only 6.98 and three of the four capacitor values are within 0.5 percent of each other. When the relative bandwidth becomes too great, it may be preferable to use separate highpass and lowpass filters in series to realize the bandpass response.

Some of the advantages and disadvantages of the 3rd-order Cauer are listed here.

Advantages

- The skirt selectivity of the Cauer is better than the Chebyshev of the same order and with other parameters being the same.
- A specific level of minimum stopband attenuation that is needed for a particular application can be specified so as to optimize the filter response for that application.
- If desired, the upper and lower stopband attenuation peaks can be manually shifted to obtain maximum attenuation at specific frequencies while maintaining an acceptable passband return loss.

Disadvantages

- Compared to the 3rd-order Chebyshev BPF, the Cauer requires four resonators—one more than the Chebyshev.
- The four Cauer resonators must be tuned to three different frequencies instead of just one as with the Chebyshev BPF, thus complicating the tuning adjustments because of the better selectivity of the Cauer.
- The required inductor Q in the Cauer BPF is higher and the component tolerances are more stringent than in the Chebyshev.
- Calculating the component values of the two series-connected resonators shown in Figure 3 is difficult.

The last item is the main disadvantage of the Cauer. However, a simple solution to this difficulty will be explained so any EMI technician or engineer will be able to easily design and evaluate any 3rd-order Cauer filter.

The main advantage of the Cauer BPF is its selectivity as indicated by the attenuation response in Figure 4 and the bandwidth parameters listed in Figure 3. For example, for the same 3-dB bandwidth, the Cauer 30-dB bandwidth is 6.4 kHz as compared to the 9.31 kHz 30-dB bandwidth of the Chebyshev. Another way of comparing these two BPFs is to compare their 30-to-3-dB shape factors. The Cauer BPF has a shape factor of 1.94, while the Chebyshev shape factor is 2.82. Although the Cauer BPF is obviously more selective than the Chebyshev, the difficulty in calculating Cauer designs has prevented the average EMI technician or engineer from using this type of filter until now.

USING ELSIE TO DESIGN A CAUER BANDPASS FILTER

Those readers having been patient enough to reach this point will be pleased to learn about an offer of free filter design and analysis software that can be used to design and evaluate any 3rd-order filter, either lowpass, highpass, bandpass or stopband. Of course, a DOS-based computer will be necessary, but virtually everybody working in electronics has a computer at home or one at work, so that should be no problem.

Those interested in passive LC filter design on an amateur or professional basis may experience the capabilities of a filter design and analysis software named "ELSIE." This software is available on a 3-1/2-inch demo disk.[6] Although the demo software is restricted to only passive filters of the 3rd order, one can still experience all the capabilities of ELSIE in the design and analysis of filters. For example, the Cauer bandpass filter shown in Figure 3 was designed with ELSIE by specifying a ripple passband bandwidth of 2.253 kHz (to give a 3-dB bandwidth of 3.3 kHz), a center frequency of 1.2 kHz, a stopband width of 6.4 kHz, a stopband depth of 30.0 dB, and input/output terminations of 192 ohms. A 30-dB bandwidth of 6.4 kHz was chosen for the Cauer so its passband return loss of approximately 20 dB would be the same as the Chebyshev BPF, thus permitting a valid comparison to be made between the stopband responses of the two BPFs.

The computer-calculated component values are listed under the schematic diagram in Figure 3. After ELSIE completed the 3rd-order Cauer bandpass design, the option of plotting the insertion and return loss responses to the monitor was used. The scale parameters were then adjusted for the most appropriate presentation and the plots on the monitor were sent to the printer. Figure 4 shows the ELSIE response plots printed on a Panasonic KX-P1124i 24-pin dot-matrix printer.

ELSIE Version. 1.11 was used with a 386SX CPU computer operating under MS-DOS Version 4.01 at 20 MHz. Although the plotting-to-monitor response of this computer is slow, the insertion and return loss responses of a 3rd-order filter design was plotted in less than 10 seconds by using only 50 data points for preliminary plots. After a satisfactory plot was obtained, 300 data points were specified for a final plot. ELSIE Version 1.11 requires less than 1 MB of hard disk space, while a more recent Version 1.23 requires about 1.1 MB. A hard disk is required to use ELSIE.

REALIZATION OF THE CAUER BANDPASS FILTER

Once the BPF design has been obtained, it is necessary to determine if it is possible to actually realize the design using commonly available components. Referring to the component values of resonators 1 and 4 listed in Figure 3, there appears to be no problem with realizing the C1 and C4 capacitor values of 0.2824 µF. This can be done by selecting two capacitors from a group of 0.27-µF capacitors so their values are within 1 percent of the 0.282-µF de-

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