Software Calculation of EM-Field Coupled Current and Voltage Levels

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OVERVIEW

The mathematics associated with radiated emissions from wires and PCB traces has been thoroughly discussed in the literature. Very little information is available, however, concerning the mathematical solution of Maxwell's equations relating to EM-field vectors that couple onto harness wires and PCB traces. Perhaps this has to do with the fact that the analysis involves tedious solutions of the equations.

The application of Mathcad to E-field coupling problems provides accurate, rapid solutions.

The Maxwell's equations that govern E-field vector coupling are complex. Many variables, such as wire length, gauge, separation, frequency, source impedance, load impedance, characteristic impedance, EM-field polarization, angle of incidence, and presence or absence of a ground plane, govern the solutions.

Mathematical implementation of the above parameters in the solution of the corresponding Maxwell's equations requires the services of a trained specialist. However, a relatively new application of Mathsoft's *Mathcad*[1] program, described in the five examples below, quickly calculates solutions to Maxwell's equations that relate to the EM-field coupling parameters listed above.

The appropriate *Mathcad* coupling file must be opened to solve the EM-field coupling calculations. This file can either be generated by the user or obtained from an outside source. When first opening the file, the "Manual/Auto" switch symbol in the toolbar must be switched to "Manual" and scrolled to the section in the coupling file containing the calculations to be performed. These are functions of E-field magnitude and coupling angle.

The input parameters corresponding to a particular set of E-field coupling conditions are entered and the Input/Auto switch is turned to Auto. The Mathcad program then calculates and plots the E-field coupled voltage and current spectra on the source and load side of the line as indicated in the five examples discussed below.

Solutions to the Maxwell's equations that give coupled voltage and current magnitude versus frequency for a variety of wire/field configurations are also available.[2]

EM-FIELD COUPLING EQUATIONS

In the five examples that follow, the EM-field vector Ez is parallel to the plane of the wires without a ground plane (Figure 1). Wire length l and separation b are given in meters. Arbitrary source and load impedances are indicated as Zs and Zl. The line characteristic impedance Zc is 100 ohms in all examples. EM field strength is 100 volts per meter and frequency f is expressed in MHZ in powers of ten to allow semilog plots in Figures 2a through 2d below.

The following equations solve for the voltage and current magnitudes associated with the E-field vector in Figure 1.

Equation (1) is the solution to Maxwell's equations, in Mathsoft format, that solves for induced load current (Il_k) in amperes for the configuration shown in Figure 1.

The terms in Equation (1) are defined by the Mathcad units. The symbol (k) is a running frequency index that allows the equations to be solved between 1 MHZ and 1 GHz.

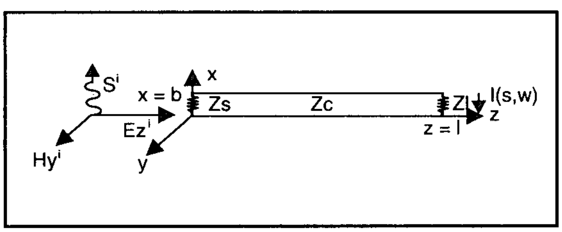

Figure 1. *E Field Parallel to Wires.*

$$IIs_k := \frac{-2 \cdot j \cdot E_Z \cdot \sin\left(\beta_k \cdot \frac{b}{2}\right) \cdot \left[Zc \cdot \sin(\beta_k \cdot 1) + j \cdot Zs_k \cdot (1 - \cos(\beta_k \cdot 1))\right]}{\beta_k \cdot \left[Zc \cdot (Zs_k + Zl_k \cdot \cos(\beta_k \cdot 1) + j \cdot (Zc^2 \cdot \sin(\beta_k \cdot 1) + j \cdot (Zc^2 + Zs_k \cdot Zl_k) \sin(\beta_k \cdot 1)\right]} \qquad (1)$$

$$dBIIE_k := 20 \log\left(\left|\frac{IIs_k}{E_Z}\right|\right) \qquad (2)$$

$$dBII_k := dBIIE_k + 20 \log\left(\left|E_Z\right|\right) \qquad (3)$$

$$dBVV_k s := dBII_k + 20 \log \left(\left| Zl_k \right| \right) \quad (4)$$

Where

$fo := 1$ = Start frequency in MHz

$f_k := 10^{\frac{k}{1000}}$ = Frequency (MHz)

$k := 0,1...3000$ = Frequency Running Index

$c := 3 \cdot 10^8$ = Speed of Light

$\beta k := \frac{2 \cdot \pi \cdot f_k}{c} \cdot 10^6$ = Wave Number

$Zs := 10^4$ $\quad l := 3$ $\quad E_Z := 100$

$Zl := 10^4$ $\quad b := .05$ $\quad Zc := 100$

Equation (2) is the induced load current ($dBIE_k$) normalized to a 1 V/m E-field vector in units of dBA/V/m. Equation (3) solves for induced load current in dBA. Equation (4) converts the induced load current to induced voltage in terms of dBV.

DISCUSSION

The current spectrum ($dBII_k$) on the right of Figures 2a, 2b, 2c, and 2d is always below the voltage spectrum ($dBVI_k$) unless only one waveform is present, due to superposition. This occurs when Zs or Zl = 1 ohm.

The Mathcad calculated wire-induced voltage levels in Figures 2a through 2d were also empirically measured with an RF voltage probe and optical link.[3] Probe error was between 3 and 5 percent. The empirical RF probe E-field vector voltage spectrum is indicated in Figure 3b.

The source and load impedances in Figure 2a are matched to the 100-ohm characteristic impedance (Zs) of the line in Figure 1. Anti-resonant points exist at each 100-MHz multiple of the full wave resonant frequency 100 MHz. This situation minimizes radiated susceptibility. The maximum E-field vector-induced voltage and current waveforms from the 100-V/m E-field vector occur in the region of 50 MHz. The peak values are 4 V and 40 mA respectively. No resonant peaks are present in this matched line.

Figure 2b indicates the voltage and current spectra at the load end of a mismatched line where Zs = 1 ohm and Zl = 10^4 ohms. This impedance combination creates the conditions for quarter wave resonance having 100-V peaks that start at 25 MHz.

This set of conditions produces unacceptably large input voltage across high impedance devices at the load end of the line.

The spectra in Figure 2c occur at the source end of the wire in Figure 2b under the same set of coupling conditions.

In Figure 2d the source and load impedances are both 1 ohm. The voltage and current spectrum plots, in this instance, are relatively flat and are without resonances. The induced voltage in magnitude is approximately 1 V, and load current is approximately 0.1 A.

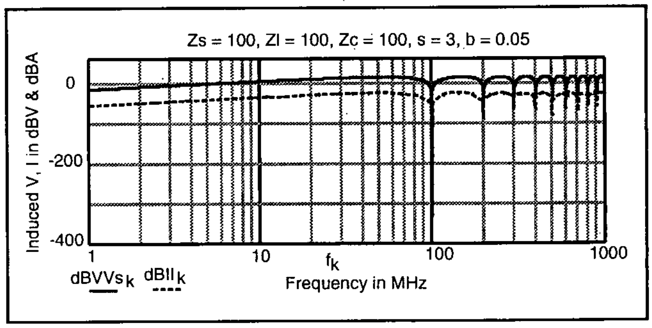

Figure 2a. *Zl and Zs Matched to Zc.*

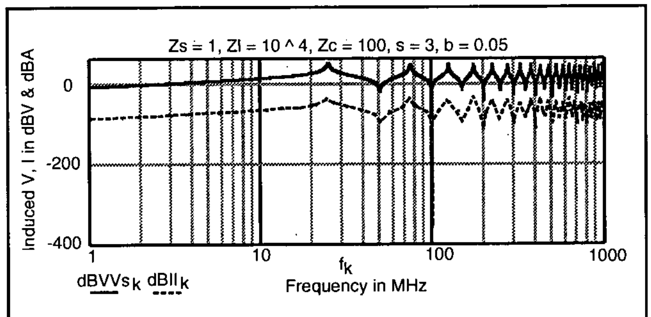

Figure 2b. *Zs = 1 ohm and Zl = 10^4 ohms (Load End Spectra).*

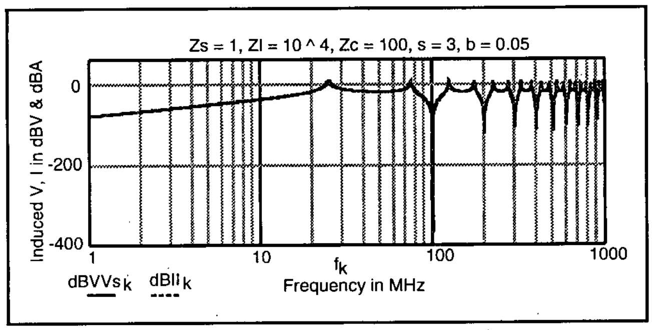

Figure 2c. *Coupling at Source End of Wires Given in Figure 2b.*

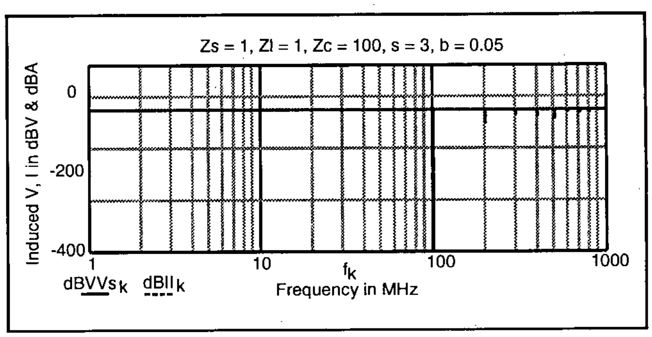

Figure 2d. *Zs = Zl = 1 ohm.*

AN HCMOS APPLICATION

A line having the dimensions shown in Figure 3a is located between two typical HCMOS "NOR" gates. The output impedance (Zs) of the left gate was measured as 100 ohms in parallel with 20 picofarads (pF) of capacitance.

The input impedance of the right gate was measured as 5 pF in parallel with 10^{12} ohms. Both impedances were measured

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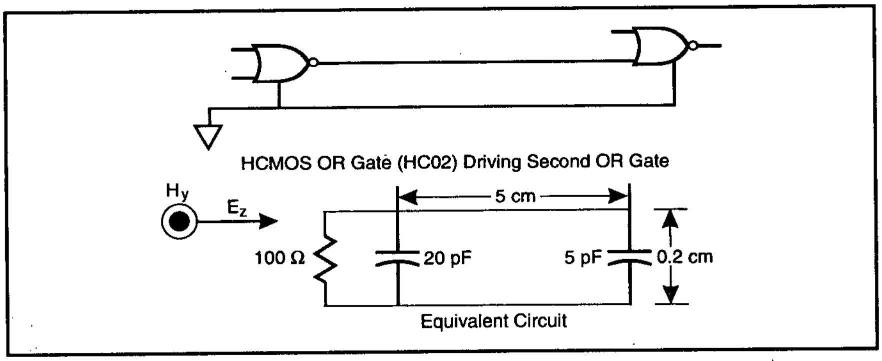

Figure 3a. *Coupling onto PCB Traces Between Logic Gates.*

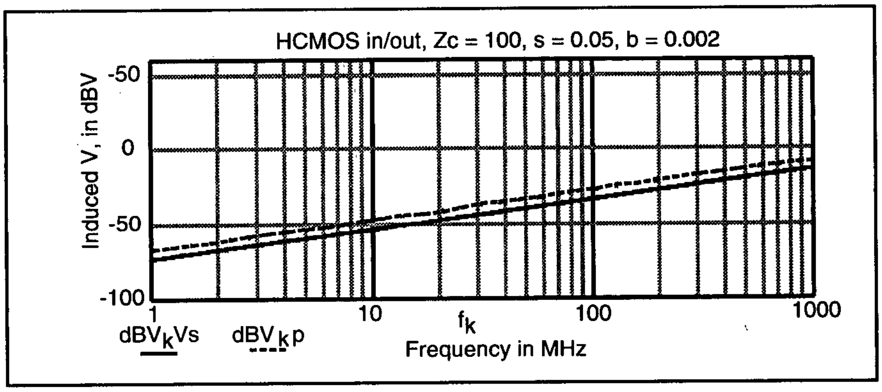

Figure 3b. *Induced Voltage in dBs for Configuration of Figure 3a.*

using the Fast Accurate Impedance Measurement (FAIM) software system.[4]

The bottom spectrum in Figure 3b is a Mathcad-generated plot of the E-field induced voltage spectrum in dB volts. The top spectrum is this induced voltage measured with an RF probe.[3] The plot indicates no coupled voltage resonances on the 5-cm line which, in this instance, is less than 1/4 wavelength long at 1 GHz. The maximum induced voltage is 0.18 volts (-15 dBV).

The lack of calculated and measured voltage resonances in Figure 3b verifies that electrically short lines, (line length less than 1/4 wavelength long at the highest frequency of interest) have very low induced voltage and current levels.

CONCLUSION

A comparison of the induced voltages with known susceptibility thresholds for various types of analog and digital electronic devices allows predictions of radiated susceptibility to be made.

REFERENCES

1. Mathsoft, *Mathcad*, 101 Main St., Cambridge, MA.

2. A.A. Smith, Jr., *Coupling of External Electromagnetic Fields to Transmission Lines*, Interference Control Technologies (ICT), 1987.

3. W. Rogers, "A Software Program for Characterizing and Designing EMI Filters." *ITEM*, 1992, pp. 16-24.

4. W. Rogers, "Advancements in Locating Susceptible DUT Wires," *ITEM* 1993, pp. 198-206.

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