

Application of the LT-MP Principle to the Theory of Lightning Propagation

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The Least Time - Maximum Probability Principle is a selection rule for finding the most probable location of a lightning strike on a structure.

INTRODUCTION

For more than 300 years, the principle of least time/least action was successfully applied by individuals like Fermat (to derive Snell's Diffraction Law in optics), Hamilton (to derive the geodesic equations for bodies in motion), Lagrange (to formulate generalized force equations), Dirac/Feynman (to derive Schrodinger's Equation), and many others.

This article begins with a fundamental scientific hypothesis and proceeds to introduce the Least Time - Maximum Probability (LT-MP) Principle and a fundamental effectiveness coefficient. The LT-MP Principle is Fermat's Principle of Least Time and classical probability¹ theory combined into one. The effectiveness coefficient compares the probability of different sets of possible outcomes of an observation. They are the cornerstones of all scientific theories, such as the LT-MP Theories of Lightning Propagation and Lightning Protection Systems Design, and the LT-MP Theory of Relativity (See Appendix B).

The LT-MP Principle encompassed by the Theory of Lightning Propagation is a selection rule for finding the most probable channel in cloud-to-ground lightning, and, therefore, for finding the most probable location where lightning is going to strike a structure, as well as the angle of incidence measured between the lightning channel and a structural element at the point of contact. The theory

explains that the tortuous path and branching of a step leader are the result of the existence of domains of relative homogeneity in the atmosphere.

The effectiveness coefficient in the Theory of Lightning Protection Systems reveals the existence of Lightning Zones, including the illusive "Lightning Protection Zone." The results will be compared with conventional models presented by Golde² and in The Lightning Protection Code in NFPA 780. This coefficient quantifies the quality of a lightning protection system, or its ability to attract lightning onto itself, thereby diverting it away from nearby structures.

The unit effectiveness coefficient (UEC) plays a particularly important role in all LT-MP theories. For example, in the Theory of Lightning Protection Systems Design, the UEC defines the boundary of a lethal zone; in the Theory of Lightning Propagation, it defines the leading edge of the traveling electron cloud in a lightning stroke; in the LT-MP Theory of Special Relativity, the UEC is the Equivalence Principle.

Appendix A shows that a theory of lightning propagation based on Ohm's Law (Least Impedance) is equivalent to one based on Fermat's Principle (Least Time).

Appendix B revisits the Michelson-Morley experiment and substantiates the claim that the LT-MP Principle and the effectiveness coefficient are both fundamental concepts. In the beginning only Einstein accepted the results

of the Michelson-Morley experiment at face value: he also had the courage to postulate against all common sense that the speed of light in a vacuum is constant and the same for all observers, irrespective of their motion. The LT-MP Theory shows the reason why Einstein's postulates in the Theory of Special Relativity are true.

THE SCIENTIFIC HYPOTHESIS THE ORIGIN OF THE LT-MP PRINCIPLE

The origin of the Least Time - Maximum Probability principle can be traced back to a fundamental scientific hypothesis. This hypothesis is phrased as follows: in any frame of reference where the total number of possible outcomes of an observation is N , the number of similar outcomes n is inversely proportional to the average time τ needed to complete the process that generates the observed outcome.

The scientific hypothesis contains two classical and fundamental concepts. They are:

- The proportionate number of similar outcomes (n/N): it is the classical definition of the probability of an observed outcome.
- The average time τ needed to complete the process that generates the observed outcome. In a process involving a group of particles, the inverse of the mean time ($1/\tau$) measures the frequency of occurrence of the observed outcome.

Thus, the scientific hypothesis is a simple statement of a generally known fact: The probability of the outcome of an observation P is directly proportional to the frequency of occurrence of the observed outcome.

THE LT-MP PRINCIPLE

The parametric equation for the scientific hypothesis can be written as follows:

$$P(p) = \frac{1}{N} \cdot \left(\frac{1}{\tau(p)} \right) = \frac{\left(\frac{1}{\tau(p)} \right)}{\sum_{\text{all } i} \left(\frac{1}{\tau_i(p)} \right)} \quad (1)$$

where N is the total number of possible outcomes. Equation (1) clearly shows that the maximum probability is inversely proportional to the least time. This is the Least Time - Maximum Probability (LT-MP) principle, given by:

$$P_{\max}(p) = \frac{1}{N} \cdot \left(\frac{1}{\tau_{\min}(p)} \right) \quad (2)$$

All scientific theories are governed by this principle. In this article, the principle is applied to the study of cloud-to-ground lightning. To demonstrate its fundamental nature, the principle is applied to derive the LT-MP Theory of Special Relativity. As expected, the LT-MP Theory of Special Relativity and Einstein's Theory of Special Relativity agree. The difference is that the LT-MP Theory is a statistical theory that accommodates the uncertainty principle in quantum physics.

LIGHTNING PHENOMENOLOGY

The LT-MP principle is a selection rule for finding the main lightning channel. According to the scientific hypothesis, the time needed to complete the process that yields the observed outcome is a key parameter in any scientific theory. In lightning, the observed outcome is a luminous lightning channel, and the process is the transport of an ensemble of charged particles from a source region (also called its origin) to a drain region (also called its destination). Lightning follows many channels, and by definition, the main channel is the channel occupied by the majority of charged particles. Thus, the LT-MP Theory of Lightning Propagation states: Lightning follows the path of least time because it is the path of maximum probability.

In any given medium, the time to transport a charged particle is the distance traveled divided by its velocity in that medium. Any ensemble of particles is characterized by a velocity distribution of its constituents, and in a given medium the majority of the particles travel at an average velocity, $\langle u_m \rangle$.³ Therefore, for the majority of charged particles in an ensemble, the average transport time τ is:

$$\tau(p) = \frac{\text{Air Channel}}{\langle u_o \rangle} + \frac{\text{Structure Channel}}{\langle u_s \rangle} \quad (3)$$

where $\langle u_o \rangle$ and $\langle u_s \rangle$ are the average velocities in air, and in a conducting structure, respectively. By applying the variational principle to Equation (3), an expression for the most probable path of a lightning channel is obtained. Thus, the LT-MP Principle is a selection rule for finding the main, i. e., the *most probable* lightning channel. These concepts are now applied to examine cloud-to-ground lightning.

CLOUD-TO-GROUND LIGHTNING

Let us assume that for the last step of a step-leader of a lightning stroke there is a source region near a tower on flat terrain at an altitude ζ above the ground (Figure 1). Lightning may strike either the ground or the tower. The minimum time for a direct strike to ground is:

$$\tau_o(\zeta) = \frac{\zeta}{\langle u_o \rangle} \quad (4)$$

The corresponding parametric equation for a deflected stroke through the tower is:

$$\tau_s(p) = \frac{\sqrt{[a_h(p)]^2 + [a_v(p)]^2}}{\langle u_o \rangle} + \frac{S_{EQ}(p)}{\langle u_s \rangle} \quad (5)$$

where the horizontal and vertical components of the air channel, a_h and a_v , respectively, are shown explicitly, and $S_{EQ}(p)$ is the parametric representation for the equivalent electrical length of the structure.⁴

THE EFFECTIVENESS COEFFICIENT DEFINITIONS FOR LIGHTNING ZONES

The effectiveness coefficient compares the probability of a direct strike to two different structures "j" and "k", and is actually a renormalization procedure:

$$E_k = \frac{P_j(p)}{P_k(p)} = \frac{\tau_k(p)}{\tau_j(p)} \quad (6)$$

For example, if for the purpose of reference, "k" is chosen to be the ground, then $E_{jk} = E_{jo}$, and the range of the values for the effectiveness coefficient defines the lightning zones, i.e.,

- A structure "j" is said to be in the "lethal zone," if $E_{jo} > 1$.
- A structure "j" is said to be in the "safe zone," if $E_{jo} < 1$.
- The boundary of the lethal zone is defined by $E_{jo} = 1$.

GEOMETRICAL REPRESENTATIONS OF LIGHTNING ZONES

As stated above, the unit effectiveness coefficient (UEC) defines the boundary of the lethal zone:

$$E_{so} = \frac{\tau_o(p)}{\tau_s(p)} = 1 \quad (7)$$

If one substitutes Equations (4) and (5) for τ_o and τ_s , respectively, into Equation (7), one obtains an equation of an ellipse:

$$\left(\frac{\rho}{\rho_x}\right)^2 + \left(\frac{z - z_x}{z_x}\right)^2 = 1 \quad (8)$$

where the semi-major axis, z_x (also called the critical height), and the semi-minor axis ρ_x (also called the critical range), are given by:

$$z_x = \frac{\zeta}{(1 + \cos\beta)}; \text{ and } \rho_x = z_x \sin\beta \quad (9)$$

The critical angle of incidence at the attachment point on a structure β is given by:

$$\cos\beta = \frac{u_o}{u_s} \quad (10)$$

Equation 8 is the expression for the "critical ellipse." The lethal zone is the solid of revolution of the critical ellipse about the vertical axis. Any structure that by design, or by accident becomes a preferred lightning target is called a primary structure: a lightning protection system (LPS), for example, is a primary structure by design. A structure that is not struck by lightning, because of the presence of other intentional, or incidental structures is called a secondary structure. Protected ground installations, for example, are called intentional secondary structures. Each primary structure has a lightning protection zone (LPZ) associated with it. An attribute of a LPZ is that during the time when a lightning source region is within the critical range ρ_x of the primary structure, the transport time to the primary structure τ_1 is always less than the transport time to either the secondary structure τ_2 or to ground τ_o , i. e.,

$$\tau_1 < \tau_2 < \tau_o \text{ or } \tau_1 < \tau_o < \tau_2 \quad (11)$$

For a grounded Franklin rod, the LPZ is the solid of revolution of a quarter segment of a tangent critical ellipse about the axis of the primary structure (TK in Figure 2). When the source region is outside the critical range of a structure, that structure will be safe from a direct strike, because it is beyond the striking distance of a lightning stroke. The LT-MP lightning zones are shown in Figure 2.

Figure 3 compares the LT-MP protection zone for a grounded lightning rod with conventional lightning protection zones. The latter are described in the Lightning Protection Code in NFPA 780, and in the international lightning code in IEC 1024. Of special interest in Figure 3 is the figure of an unprotected rocket. If one were asked to assess its protection on the basis of the Lightning Protection Code in NFPA 780, one is expected to conclude that the rocket is protected with adequate safety margin. In Appendix A, it is shown that the LT-MP Theory of Lightning Propagation and Ohm's Law are equivalent. The Rolling Ball Theory is shown to violate Einstein's Theory of Special Relativity and

Ohm's Law. It can therefore be concluded that the Rolling Ball Theory, and thus the Lightning Protection Code, is wrong. For the next update of the protection code, it is strongly recommended that all lightning protection systems comply with the LT-MP Theory of Lightning Protection Systems, i.e., with Ohm's Law.

A PASSING LIGHTNING STORM

Figure 4 shows a sequence of lightning strokes from a storm passing over a tower on flat terrain. The LT-MP Theory predicts that distant electrical storms will discharge directly to the ground. When the storm reaches the critical range ρ_x the probability of a direct strike to an erect tower or to the ground is equal. At this range the effectiveness coefficient is one ($E_{so}=1$). With the source

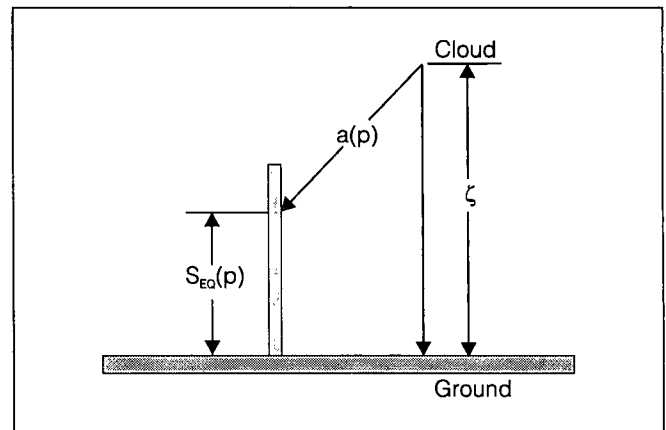


Figure 1. Cloud-to-ground Lightning Near a Grounded Structure.

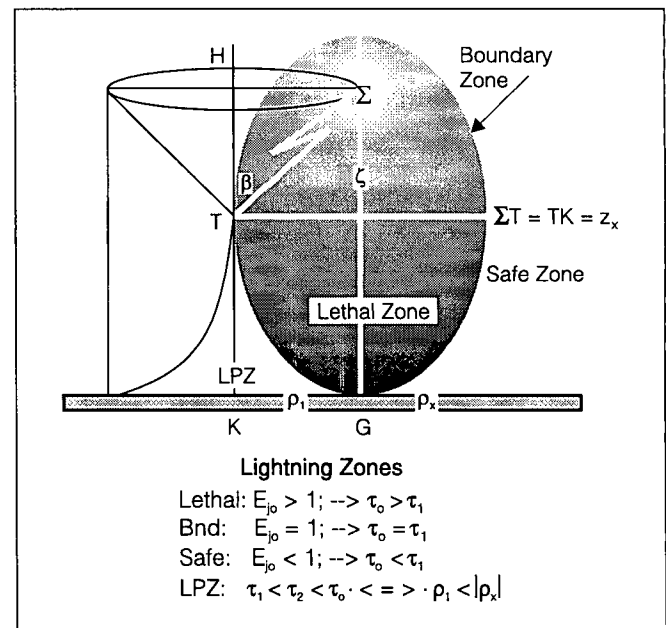


Figure 2. The Geometry of Lightning Zones.

region inside the range of attraction, $\rho_h < \rho < \rho_x$, the storm is said to be inside the attraction cone, and as the source moves closer to the tower, the point of attachment on the tower will move up until it reaches the top of the tower. This occurs when the storm reaches the range ρ_h , where it enters the tracking cone. Lightning will continue to strike the highest point of the tower until the source region leaves the tracking cone and re-enters the attraction cone. Lightning strikes the tower between its highest point and the critical height z_x as long as the source region remains inside the cone of attraction. The tower comes out of the lethal zone when the source region leaves the attraction cone.

TORTUOSITY AND BRANCHING OF LIGHTNING CHANNELS

It is Ohm's Law acting in non-homogeneous air that makes lightning do what it does! With each stroke, lightning reveals detailed information about the structure of the conducting air mass in a specific region at a specific time. The LT-MP Theory of Lightning Propagation explains that lightning follows a tortuous path because the air mass in the atmosphere has contiguous domains of relative homogeneity, separated by boundary layers where changes in the electrical properties of the air mass occur rapidly over short distances. A step-leader changes direction when it crosses a boundary between two domains. The angle of refraction, also called the angle of deflection, is governed by Equation (10) which is the analog of Snell's Law, i.e.,

$$\cos \beta_{12} = \frac{\langle u_1 \rangle}{\langle u_2 \rangle} \quad (10a)$$

Natural lightning also shows frequent branching. This occurs when the transport times in nearby channels are of comparable duration. When this occurs, the ensemble of charged particles distributes itself over the available channels in proportion to the probabilities given by Equation (1). This is the analog of Ohm's Law for parallel resistors, and, thus, the LT-MP Theory of Lightning Propagation can be restated as follows: Lightning follows the path of least impedance, because it is the path of maximum probability.

"Lightning" of course, is the same as "electricity." The second part of the definition of Ohm's Law in the statement above is due to the LT-MP Principle. It answers the question that for many years bothered this and other authors: How does an electron know which is the path of least impedance? With the possible exception of The Feynman Lectures on Physics, broadly interpreted, this author is not

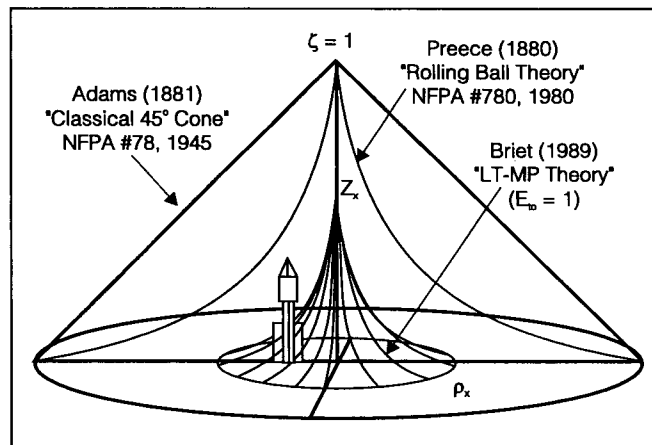


Figure 3. Comparison of Lightning Protection Zones. Note the exposed rocket.

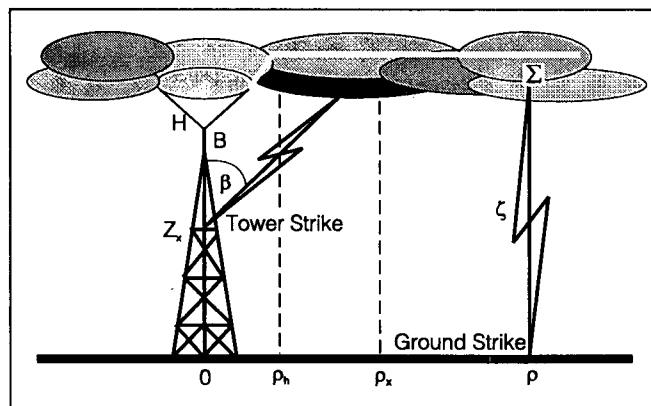


Figure 4. Assumptions: Flat terrain, constant ζ ; homogeneous ground. Distant lightning strikes the ground below the source region. When the source region is inside the attraction cone, lightning will strike the tower at a point between the critical height and its peak. Lightning will strike a tower at its highest point when a source region is in the tracking cone.

aware of other references where Ohm's Law is presented with a statistical explanation of the behavior of electricity/lightning.

FURTHER OBSERVATIONS

The surface of the sphere in the Rolling Ball Theory (RBT) is a boundary of a lethal zone: it is defined by $E_{s0} = 1$. For a UEC, the transport time of a lightning stroke through a tangent structure $[\tau_t = (a/u_o) + (S_{EQ}/u_s)]$ must be equal to the transport time for a direct stroke to the ground $[\tau_o = (\zeta/u_o)]$ (Figure 5). Since radii a and ζ are equal, this implies that the

*The Rolling Ball Theory violates the
Theory of Special Relativity and Ohm's Law.*

travel time and the resistance (or the impedance) in the structure must both be zero. For the first condition to be true, the RBT requires the average velocity of lightning in a structure to exceed the velocity of light in a vacuum. This violates Einstein's Theory of Special Relativity. Equivalently, one can argue that Ohm's Law (See Appendix A) is violated: the RBT requires that the resistance in all tangent lightning intercepting structures be zero. It is concluded that the RBT is wrong, because it violates basic laws of physics. This is a compelling reason why the Lightning Protection Code in NFPA 780 must be updated.

The natural recommendation to update the Lightning Protection Code is to add at least a requirement that all lightning protection systems be designed in a way that is demonstrably consistent with Ohm's Law. This can be achieved by applying the LT-MP Principle, which is the equivalent of Ohm's Law (See Appendix A).

What will happen if the code is not updated? Of course, lightning protection systems will continue to fail. In most cases, the consequence of a failing protection system can be ignored. However, the risks from induced or direct lightning strikes can not be ignored if the protected assets happen to be nuclear power plants, rocket launch complexes, local or global communications centers, or other critical installations, including tank farms that shelter hazardous materials or deployed systems that employ sophisticated, but vulnerable electronic equipment. For critical installations maximum lightning protection should be mandated.

HISTORICAL BACKGROUND OF THE LT-MP PRINCIPLE

Some readers may think that the LT-MP principle is a brand new idea that has not yet been verified; nothing is further from the truth. For many years electrical and electronic equipment have, and continue to operate successfully because of Ohm's Law and the Principle of Least Time. In fact, the LT-MP Principle is at least two thousand years old.⁵ To show this, a few individuals who in the last two millennia contributed to the body of scientific knowledge that is applicable to this paper are mentioned (see sidebar). Their names and dates serve as markers for the milestones along the evolutionary highway into our time. Of course, there are many others who deserve to be mentioned, and even more whose thoughts and ideas led to the scientific hypothesis, from which the LT-MP Principle and the effectiveness coefficient evolved.

In almost every field of physics variational principles are used to derive "equations of motion." Using the variational principle as the basis for the theory of lightning propagation, there is a structural analogy with Newton's Equations, Maxwell's Equations, and the Schrodinger Equation. The author acknowledges that even the idea of making a connection with classical statistics is not new. Born's statistical interpretation of quantum theory expresses essentially the same idea. And Max Born thought of this almost 70 years ago!

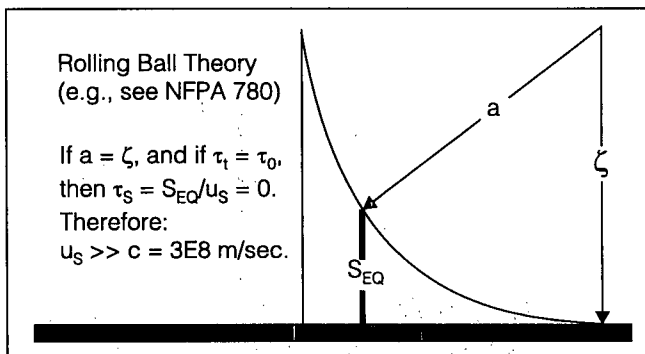


Figure 5. Lethal Zone in the Rolling Ball Theory.

SOURCES FOR THE DEVELOPMENT OF THE LT-MP THEORY

Hero of Alexandria (ca. 50 AD)

Hero discovered the principle of shortest distance. This principle explains light reflections off mirror surfaces.

Fermat (1601-1665)

Fermat discovered the principle of least time. Fermat's successful derivation of Snell's law of light refraction at an interface between two dissimilar materials lent credibility to this principle.

Euler (1707-1783), and Lagrange (1736-1813)

They discovered the variational principle, i.e., the principle of least action. Hamilton's principle, and the Lagrange Equations are based on this principle.

Born (1882-1970)

Max Born discovered the statistical meaning of quantum physics. Born's statistical interpretation of the Schrodinger Equation enabled modern physicists to begin to understand the strange theory of quantum physics.

Dirac (1903-)

Dirac made many contributions in mathematical and quantum physics. Dirac's paper on "The Lagrangian in Quantum Mechanics" in the *Physikalische Zeitschrift der Sowjetunion*, 1933, relates to the material presented in this article.

Feynman (1918-1988)

Starting with Dirac's '33 paper, Feynman derived the Schrodinger Equation. Although Feynman originally did not like the Least Action Principle, he changed his mind and used the principle extensively after the mid-forties.

What makes this article unusual is the introduction of the effectiveness coefficient E_{jk} given by Equation 6. This parameter facilitated the discoveries of the lethal, protection, and safe zones associated with lightning.

The significance of the effectiveness coefficient reaches far beyond the theory of lightning propagation. The unit effectiveness coefficient (UEC) in particular plays a central role in many physics theories. In Appendix A, for example, the UEC describes the leading surface of a lightning stroke as a shock wave traveling at the average velocity, $\langle u_o \rangle$, in air. In Appendix B the UEC is the Equivalence Principle in the derivation of the transformation equations for inertial reference frames in motion relative to one another. The derivation yields the same Lorentz-Fitzgerald contraction obtained by Einstein in his Theory of Special Relativity. For motion along the x-axis, the Lorentz-Fitzgerald contractions are:

$$\text{Time dilation: } t_m = t_s \sqrt{1 - \left(\frac{V}{C}\right)^2}$$

$$\text{Length preservation: } y_m = y_s$$

$$\text{Length contraction: } x_m = x_s \sqrt{1 - \left(\frac{V}{C}\right)^2}$$

SUMMARY AND CONCLUSIONS

A degenerate statement of the scientific method written in the style of Descartes⁶ is: "It works, therefore it is." It is an expression more of attitude than a dogma or a principle. It is comfortable, because it permits one to avoid questions like: "How does a ball know which path to choose?" (Lagrange's geodesic equations); "How does a photon know when, and how much to turn, as it enters a slab of glass?" (Snell's Diffraction Law); or "How does an electron know which is the least resistance path?" (Ohm's Law).

Feynman believed he found the answer in quantum mechanics. The LT-MP Principle goes straight to the core of the scientific method and answers these questions in the same way that Born and Feynman did. Born introduced the statistical interpretation of quantum mechanics. Now the statistical interpretation of the Least Time/Action Principle, with an effectiveness coefficient added, is introduced. The effectiveness coefficient "renormalizes" the striking probability distribution function: In one interpretation it generalizes the Equivalence Principle.

The LT-MP Principle and the effectiveness coefficient enables one to recognize old things from a new perspective. Admittedly, not all LT-MP Theories will immediately yield new results,⁷ but sometimes, as it did for the LT-MP Theory of Lightning Propagation and the LT-MP Theory of Lightning Protection systems design, they do provide new insights. The LT-MP lightning theories solved many riddles about lightning.

For the reader in search of practical applications, this article has shown that effective lightning protection systems can and should be designed within the bounds of mainstream science.

APPENDIX A: CORRESPONDENCE BETWEEN OHM'S LAW AND FERMAT'S LEAST TIME PRINCIPLE

The objectives of this appendix are to:

- Show the one-to-one correspondence between Ohm's Law of Least Resistance/Impedance and Fermat's Principle of Least Time.
- Show how the electric field dependence is accounted for in the LT-MP Theory of Lightning Propagation.
- Show that the effectiveness coefficient in the LT-MP Theory can tell us something about lightning propagation.

The theory of lightning propagation was developed on the basis of Fermat's Principle of Least Time. Without justification, it was claimed that the propagation of lightning can be described by either one of two equivalent statements.

- Lightning follows the path of least time, because it is the path of maximum probability.
- Lightning follows the path of least impedance, because it is the path of maximum probability.

That claim will now be justified. Using Jordan and Balmain's book as an arbitrary reference, we start with Ohm's Law where the following expression is given (Jordan and Balmain, p. 205):

$$J = \sigma \cdot E \quad (A-1)$$

where the current density $J = ne\langle u \rangle$ and the conductivity of the medium is $\sigma = (1/\rho)$. The electron charge density is given by $ne = dQ/dVol = dQ/(A \cdot ds)$, where A is the cross section of the lightning channel, $\langle u \rangle$ is the average velocity of the charged particles (Jordan and Balmain, p. 77) and $ds = \langle u \rangle \cdot dt$ is the distance traveled along the electric field, E (Figure A1). The electric field E is responsible for sweeping the charges from a source region to a drain region through a voltage drop V . It is given by $E = -\Delta V = -(dV/ds)$. Therefore, Ohm's Law can be written in the form:

$$\frac{dQ}{A} \cdot \frac{\langle u \rangle}{ds} = \frac{1}{\rho} \cdot E = -\frac{1}{\rho} \cdot \frac{dV}{ds} \quad (A-2)$$

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