Equivalent Time in Varistor Applications

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THE CONCEPT OF EQUIVALENT TIME

In many circumstances, the duration of a pulse is sufficiently short that essentially all of its energy is converted to internal energy with a rise in temperature proportional to the specific heat of the materials carrying the pulse currents. This is valid whenever the pulse duration is short in comparison to the thermal diffusion times. It is then convenient to define a pulse equivalent time which is reached according to the following considerations.

Equivalent time is a universal parameter associated only with the pulse shape, not its amplitude.

For a pulse initiated at t = 0, the total energy delivered to a component is given by

$$E = \int_0^{\infty} i(t)\, V(t)\, dt$$

If the component under consideration is a surge arresting device, for pulses of large amplitude the one of interest, the voltage across the device, will be substantially constant during the most significant portion of the pulse event. In that case, the integral above becomes

$$E = V_c \int_0^{\infty} i(t)\, dt$$

where V_c is the clamp voltage of the arresting device. Further, if the pulse were rectangular, the result would be

$$\int_0^{\infty} i(t)\, dt = i_p T$$

where i_p is the peak current and T is the pulse duration. An equivalent time for a pulse of arbitrary shape is then defined by

$$T_{eq} = \frac{1}{i_p} \int_0^{\infty} i(t) dt = \frac{1}{i(t_p)} \int_0^{\infty} i(t) dt$$

Note that from the last equation, the equivalent time is independent of the pulse amplitude. It depends only on the pulse shape.

For a pulse of arbitrary shape the result is

$$E = i_p V_C T_{eq}$$

Thus the equivalent time is the time duration of a constant pulse at current i_p that delivers the same energy as the pulse of arbitrary shape. Frequently a peak power is specified rather than a peak current. In that case the peak current is adequately given by

$$i_p = \frac{P_p}{V_c} \text{ and } E = P_p T_{eq}$$

Since the waveforms are substantially standardized, for any one of them the equivalent time may be calculated once and for all, and energies delivered to devices are then calculated simply from the last three equations.

THE EXPONENTIAL PULSE

To illustrate the principles as simply as possible, consider a single exponential pulse that is zero for negative times and for positive times is given by

$$i(t) = i_o e^{-\frac{t}{\tau}}$$

In this case, the peak current is i_o and the equivalent time is given by

$$T_{eq} = \frac{1}{i_p} \int_0^{\infty} i_o e^{-\frac{t}{\tau}} dt = \int_0^{\infty} e^{-\frac{t}{\tau}} dt = -\tau e^{-\frac{t}{\tau}} \Big]_0^{\infty} = \tau$$

For the single exponential, the equivalent time is the exponential time constant. In one sense, that is a rather remarkable result, and is a special property of the exponential function. Why it is remarkable is illustrated by Figure 1.

Our result says that the area under the infinitely long tail of the exponential is exactly equal to the area above the curve and contained in the unit rectangle.

THE DOUBLE EXPONENTIAL

A type of pulse frequently of interest is shown in Figure 2.

It is characterized by the time it takes to reach the peak and the time it takes to reach half amplitude beyond the peak. The specification is generally written as t_1 x t_2, where both times are measured

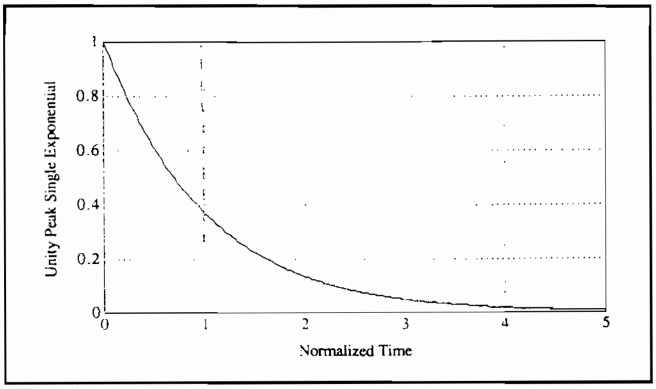

Figure 1. *Single Exponential Pulse.*

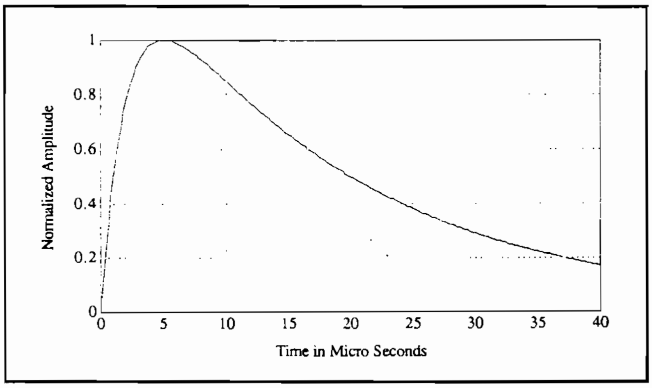

Figure 2. *5 x 20 μs Pulse.*

from the time origin. The pulse displayed is a unity amplitude 5 x 20 μs pulse. These times were chosen to clearly exhibit the general pulse characteristics. The specific example is a double exponential whose analytic representation is

$$i(t) = i_0 \left[\exp\left(-\frac{t}{\tau_d}\right) - \exp\left(-\frac{t}{\tau_r}\right) \right]$$

where τ_d is the exponential decay time constant and τ_r is the exponential rise time constant.

Integrations similar to the one performed for the single exponential give the equivalent time as

$$T_{eq} = \frac{i_0}{i_p} (\tau_d - \tau_r)$$

That's the easy part. However, in this case i_0 and i_p are not equal. Even worse, the specification is given in terms of t_1 x t_2, not the exponential time constants. A t_1 x t_2 specification means that the exponential time constants must be determined to arrive at the equivalent time.

The time constants are found by simultaneously solving the derivative equation that locates the peak and the equation that locates the half amplitude point. These equations will be referred to as the constraint equations. The constraint equations can be respectively put in the form

$$\frac{1}{\tau_d} \exp\left(-\frac{t_1}{\tau_d}\right) - \frac{1}{\tau_r} \exp\left(\frac{t_1}{\tau_r}\right) = 0$$

$$\left[\exp\left(-\frac{t_2}{\tau_d}\right) - \frac{1}{2} \exp\left(-\frac{t_1}{\tau_d}\right)\right] - \left[\exp\left(-\frac{t_2}{\tau_r}\right) - \frac{1}{2} \exp\left(-\frac{t_1}{\tau_r}\right)\right] = 0$$

and here include two simultaneous transcendental equations. Except for the trivial solution, $\tau_r = \tau_d$, there is no analytical solution; numeric computation is necessary. That is how the 5 x 20 μs waveform shown above is reached. For that case

$\tau_d = 18.6\ \mu s$ $\quad \tau_r = 2.0\ \mu s$ $\quad i_o = 1.5$ A $\quad T_{eq} = 24.3\ \mu s$

All specification requirements were exactly met and the validity of the computer programs being used was demonstrated.

Not every t_1 x t_2 specification can be met by a double exponential function. A good example is the frequently occurring 8 x 20 μs. To see what is occurring, the constraint equations can be plotted. This is a non-trivial task, since for each point of each plot one of the transcendental constraint equations must be solved. The plot for 5 x 20 μs is shown in Figure 3.

To get a useable double exponential pulse, the two curves must cross in the region where τ_r and τ_d are not equal. Since both constraints possess the trivial solution, they are seen merging into those solutions in the region to the right of the plot. For those solutions, the double exponential is the constant zero function, so it is of no use. For the 8 x 20 μs there is no intersection in the region for nonequal τ_r and τ_d. There is no 8 x 20 μs double exponential (Figure 4).

Fortunately, an approximate equivalent time is valid for practical purposes. Such a pulse is shown in Figure 5.

What one finds here is that the exponential time constants are equal, to about three significant figures, and the value of i_o is on the order of 10^3. Thus, the computations must be done correspondingly accurately to obtain a valid result for equivalent time. The result is

$i_o = 6.1$ kA $\quad \tau_r = 7.4\ \mu s$ $\quad \tau_d = 7.4\ \mu s$ $\quad T_{eq} = 20.2\ \mu s$

The approximating waveform is actually a 7.5 x 20 μs. Plots of the constraint equations are shown in Figure 6.

It is not entirely clear from the plot, but the two curves just barely cross in the region for nonequal τ_r and τ_d where the two are nearly equal, as computed above. Estimates made with the corresponding equivalent time will be conservative because a 7.5 x 20 μs will contain slightly more energy than an 8 x 20 μs.

Given the 20 μs, one can not help but wonder if

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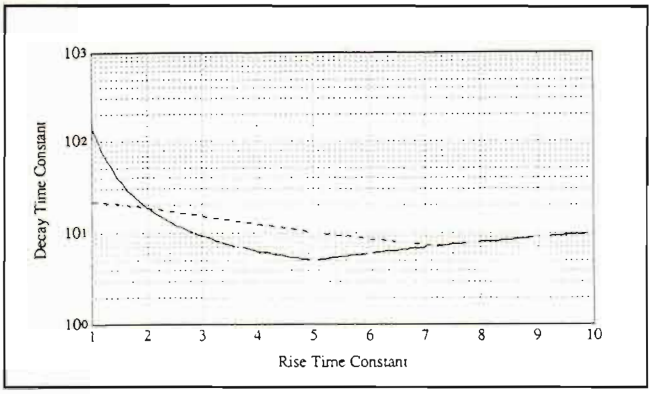

Figure 3. *5 x 20 µs Pulse. Solid line is peak constraint; dashed line is half-value constraint.*

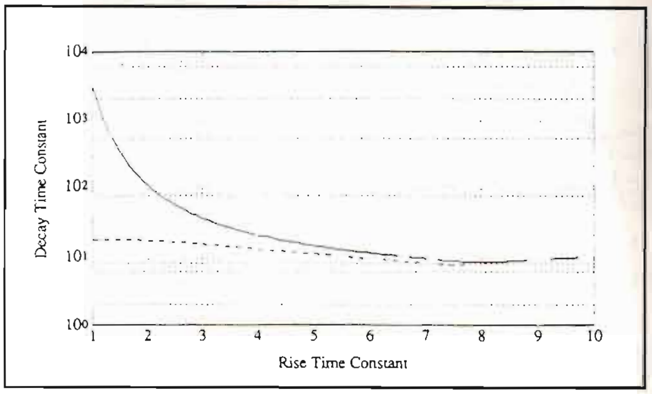

Figure 4. *8 x 20 µs Pulse. Solid line is peak constraint; dashed line is half-value constraint.*

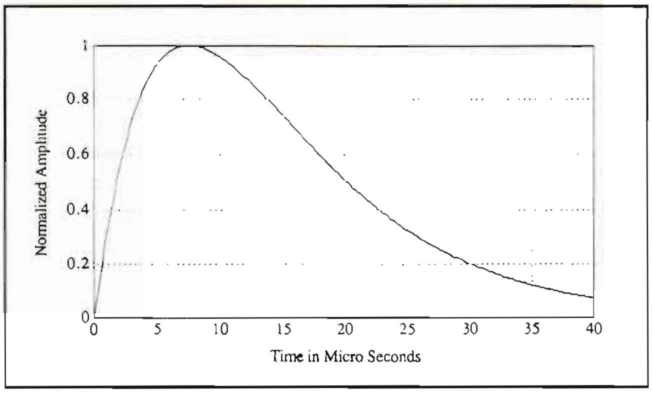

Figure 5. *Approximate 8 x 20 µs Pulse.*

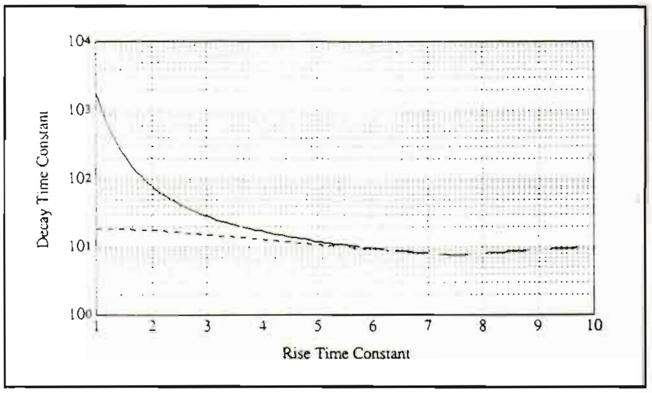

Figure 6. *7.5 x 20 µs Pulse. Solid line is peak constraint; dashed line is half-value constraint.*

someone chose the 8 µs to make this result come out the way it does. The answer is unclear. The remarkable factors are the closeness of T_{eq} to 20 µs and the fact that the resultant location is the borderline of where a double exponential can be used at all. We also note that an 8 x 20 µs waveform contains nearly the same energy as a single exponential with the same peak amplitude and a 20 µs decay time. The single exponential has an infinitely fast rise time, so can not be practically generated. The double exponential can be approximated in practice. Are these curious coincidences?

The exponentials are chosen to illustrate the concept of equivalent time because they are the simplest functions with which to deal. The difficulties alluded to in the last paragraphs were not anticipated.

THE IEC ESD PULSE

The IEC Standard 801-2, "Electromagnetic Compatibility for Industrial-Process Measurement and Control Equipment," proposes a waveform to be used for ESD testing. The waveform is given in graphical form only. Fifty-one values were read from the graph and entered into a simple computer system. A plot of the entered data is given in Figure 7.

Numerical integration was used to find the equivalent time. A number valid for practical purposes is

$$T_{eq} = 34 \text{ ns}$$

In vendor literature, test source voltages are frequently cited. To use the concept of equivalent time the peak current is needed. The IEC document specifies the conductance at peak current as 3.75 kmhos. Thus the peak current may be found by multiplying the source voltage in kV by 3.75.

LINEAR RISE WITH EXPONENTIAL DECAY

Figure 8 shows a waveform that sometimes appears in vendor literature. It has the advantage that it is analytically simple and the disadvantage that it can not be easily generated in a test system.

The double exponential waveforms are characterized by a t_1 and t_2 specification. This example is 8 x 20 µs.

Using the general methods of the first two sections, the decay exponential time constant and equivalent time are found to be

$$\tau_d = \frac{t_2 - t_1}{\ln 2} \quad \text{and} \quad T_{eq} = \frac{t_1}{2} + \frac{t_2 - t_1}{\ln 2} = \frac{t_1}{2} + \tau_d$$

For the 8 x 20 µs example, the equivalent time is 21.3 µs.

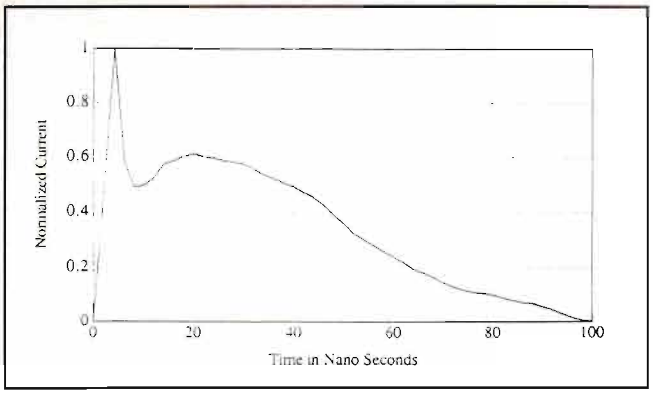

Figure 7. *Waveform Proposed in IEC Standard 801-2.*

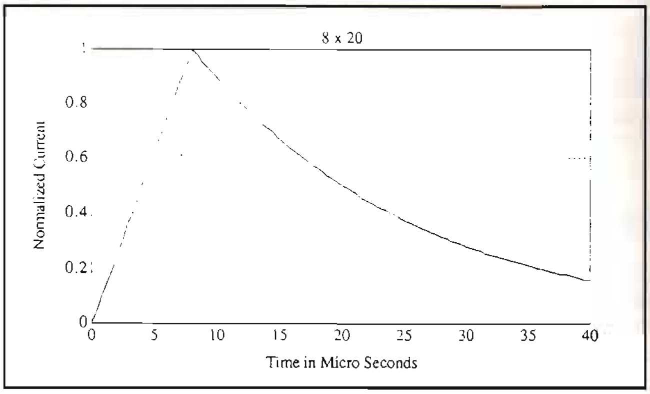

Figure 8. *Common Waveform.*

THE GAUSSIAN PULSE

In some communications receiver front-end applications, the specified pulse shape is Gaussian. The specification is given in terms of peak amplitude and 10-90% rise time, t_r. To determine the equivalent time, the peak amplitude unity normalized function will be used.

$$i(t) = e^{-\left(\frac{t}{\tau}\right)^2}$$

Since the pulse is an even function, the rise and fall times are equal, and since negative times should not be a concern, the visualization of the calculations will be made on the pulse trailing edge. t_1 will be used as the time at which the pulse has fallen to 90% of peak and t_2 as the time at which the pulse has fallen to 10% of peak. To find the equivalent time, τ must be known and to find τ, t_1 or t_2 must be known. Manipulation of the above equation gives

$$t_2 = \frac{t_r}{1-\sqrt{\ln i(t_1)/\ln i(t_2)}} \qquad t_1=t_2-t_r \quad \tau = t_2\frac{1}{\sqrt{-\ln(i(t_2))}}$$

Substituting numerical values for a 10-90% rise time gives

$$t_2= 1.272\, t_r \qquad t_1=t_2-t_r \quad \tau = 0.659\, t_2$$

The standard normalization for a Gaussian function can be expressed as

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}\mu^2}\, du = \sqrt{2\pi}$$

With the appropriate change of variables,

$$\int_{-\infty}^{\infty} e^{-\left(\frac{t}{\tau}\right)^2}\, dt = \tau\sqrt{\pi}$$

which gives an equivalent time of

$$T_{eq} = \tau\sqrt{\pi}$$

In some instances, the test methodology is in terms of a free space excitation of an antenna with the pulse amplitude specified by the open circuit peak voltage at the antenna output port. In that case, the peak current is given by

$$i_p = \frac{V_p-V_c}{Z_o}$$

where V_p is the peak voltage, V_c is the varistor clamp voltage, and Z_o is the characteristic impedance of the associated transmission line. For example, for

$V_p = 1500$ V $\quad V_c = 30$ V $\quad Z_o = 50\ \Omega \quad t_r = 3$ ns

we would have

$t_2 = 3.8$ ns $\quad t_1 = 0.8$ ns $\quad \tau = 2.5$ ns $\quad T_{eq} = 4.5$ ns
$i_p = 29.4$ A $\quad E = 3.9\ \mu$J

CONCLUSION

Frequently the total energy delivered to a varistor device is a significant factor in determining device survival. For pulses of duration short in comparison to thermal diffusion times, it is a major factor. Thus, being able to relate the total energy to other standard device specification parameters is essential. Once the equivalent time for a given pulse shape is determined, calculation of total energy in terms of those parameters, not involving the pulse shape, becomes a simple matter. The equivalent time for a number of frequently encountered waveforms is presented herein and some rather curious characteristics of the double exponential are revealed.

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