METHOD FOR DETERMINING THE GAIN AND/OR ANTENNA FACTOR OF SMALL ANTENNAS AT SHORT RANGE

The three-antenna method for determining the far-field power gains of antenna factors (AFs) of three or more different antennas without advance knowledge of the gain of any of them has been known for a long time.[1] If two of the antennas are identical and passive, the three-antenna method reduces to the two-identical-antenna method, and the minimum three antennas reduces to two. Both methods depend fundamentally upon being able to calculate accurately the so-called path loss between the sending and receiving antennas. Everything is fine in principle when the antennas are comfortably in each other's far field, but difficulties both practical and philosophical[2] in nature develop when the spacing between the antennas must be significantly less than a wavelength, and far-field gain and AF characteristics are nevertheless required to be obtained.

This article is intended to assist the engineer or technician to fully understand the three-antena method, indicate an organized procedure of application, and suggest a modification for improving the path-loss calculation to permit a more accurate determination of far-field characteristics from a near-field measurement. The approach suggested here has its limitations, but it stands nevertheless that a sensing antenna, or probe, must be calibrated as to its far-field characteristics if one is to obtain accurate measurements of signal strength, or at least of the resultant signal along the path subtended by the antenna, either in far or near-field situations.

In most TEMPEST-type measurements, for example, the receiving antennas have to be electrically small at the lower frequencies lest they subtend too much of an arc to sample the field meaningfully when placed at the prescribed usually-short distance from a source. No simple, ideal solution exists for this antenna calibration or measurement problem, but a step in the right direction is to assume that the tangential field at a given receiving antenna due to a small source, or transmitting antenna, some distance away varies much the same as that of an infinitesimal electric or magnetic dipole — that is, it involves the inverse first, second, and third powers of the radian distance between the source and the receiving antenna.

The radial field components are ignored in the following discussion, although they may be significant. The antennas are assumed to be oriented broadside for maximum transfer of energy from transmitter to receiver by the tangential field. Also ignored are mutual impedance effects, which are small for small non-resonant antennas. Under these assumptions, the amplitude of the complex tangential near-field can be said to vary inversely with ρ (defined below[3]), approaching simple inverse-distance dependence as the separation between antennas exceeds about π radians, or one-half wavelength. The asymptotic tendency of ρ toward r and r^3 for large and small separations respectively is evident in Figure 1, where 10 log ρ is plotted on semi-logarithmic coordinates as a function of the radian distance r. It is often necessary to carry out antenna calibrations and field measurements at separations far less than one radian, when the field amplitude may vary essentially as the inverse third power of the separation.

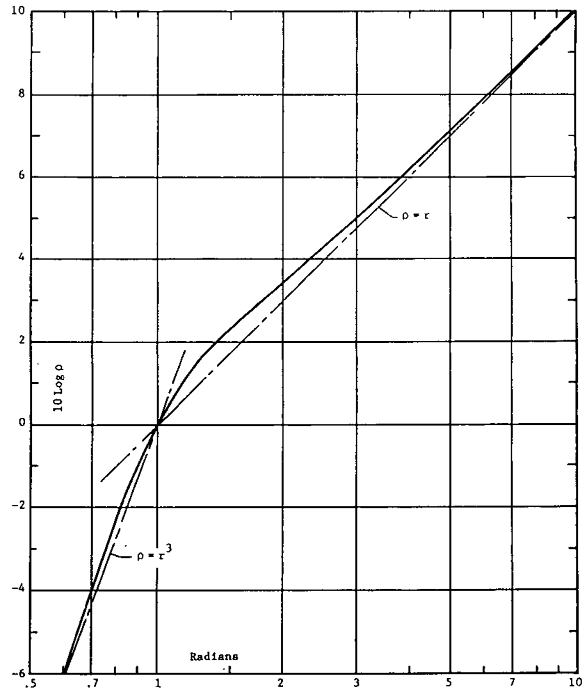


Figure 1. Plot of 10 Log ρ vs Radian Distance

The three-antenna method utilizes the third antenna to determine the product and the ratio of either the gains or the AFs, or both, of the other two antennas being tested. From this information, and a path-loss calculation, the individual gains or AFs of the two antennas are found, and hence the gains or AFs of any desired number of antennas, including the auxiliary antenna, by direct comparison with either of the original two already calibrated. The auxiliary antenna may, in fact, be one of several antennas to be evaluated if it meets the requirements specified below.

One of the antennas, say the auxiliary antenna, must be capable of transmitting. It may be either active or passive, and is not required to be capable of receiving. One of the other two, say No. 1, must be capable of both transmitting and receiving, with the same gain in either mode; that is, passive, bilateral. The remaining antenna, No. 2, must be capable only of receiving, and it may be either active or passive. If the spacing between the antennas is to be less than about π radians, antenna Nos. 1 and 2 must be of the same elementary field type and condition of electrical balance with respect to ground. Gains and AFs are determined using a fixed terminating impedance, say 50 ohms, as the antenna is intended to be used in practice, and not with a perfect impedance match obtained by laboratory tuning means. The gains are understood to be power gains relative to an ideal isotrope. It is usually most convenient to determine and express both the gains and AFs in terms of decibels. Let us define

R = Path distance *in meters* between the effective radiation centers of the antennas.

r = Radian path distance $= 2\pi R/\lambda$.

λ = Wavelength *in meters* $= 300/f$.

f = Frequency in *megahertz*.

$$\rho = \left| \frac{1}{r} - \frac{j}{r^2} - \frac{1}{r^3} \right|^{-1} = \left\{ \frac{1}{r^2} - \frac{1}{r^4} + \frac{1}{r^6} \right\}^{-1/2}$$

$$\lim_{r\to\infty} \rho = r.$$

P_t = Power radiated or transmitted.

P_r = Power received.

$p = P_r/P_t$.

Δ_{12} = 10 log p, for antennas Nos. 1 and 2 (a negative value).

G_1, G_2 = Gains relative to an ideal isotrope of antennas Nos. 1 and 2, using a fixed terminating impedance.

G(dBi) = 10 log G.

$g_{1/2} = G_1/G_2$.

$g_{1/2}$(dB) = 10 log $g_{1/2}$.

AFE = E-Field antenna factor.

AFE(dB) = 20 log AFE.

R_L = Terminating impedance.

Numerical subscripts are used to identify the antennas.

Gain

Beginning with Friis' transmission equation, one has

$$p = p_r/p_t = G_1G_2\lambda^2/(4\pi R)^2 \xrightarrow[\text{field}]{\text{near}} G_1G_2/4\rho^2 \quad (1)$$

The gain product is obtained immediately upon re-writing (1) as

$$G_1G_2 = 4p\rho^2. \quad (2)$$

By comparing antenna Nos. 1 and 2 with each other, using the auxiliary antenna, one can obtain the gain ratio $g_{1/2}$ of the two, taken as the gain of No. 1 relative to No. 2. That is,

$$G_1G_2 = g_{1/2}. \quad (3)$$

Solving (2) and (3) simultaneously for G_1 and G_2 yields

$$G_1 = 2\rho(p\, g_{1/2})^{1/2} \quad (4)$$

and

$$G_2 = 2\rho(p/g_{1/2})^{1/2}. \quad (5)$$

The unknowns, which must be determined by measurement, are p and $g_{1/2}$, and this must be done for every frequency of interest for either gain or AF. The corresponding ρ must be calculated or taken from a chart. The power ratio p becomes the insertion loss of the system when expressed in decibels as Δ_{12}, and is the attenuation required to yield the same received signal when the feeders are direct-connected through a calibrated attenuator. A fixed matching attenuator of six to ten dB should be used at the transmitting antenna input terminal to assure a matched load for the transmitter. This attenuator remains with the feeder when making the direct connection. A corresponding attenuator is not required at the receiving antenna terminal if the receiver is well matched to its feeder. The feeder system should be of the same nominal characteristic impedance throughout.

The gain ratio $g_{1/2}$ is most conveniently determined in dB as $g_{1/2}$(dB). Taking common logarithms of both sides of eq. (4) gives

$$G_1\,(\text{dBi}) = 3.010 + 10\log\rho + \Delta_{12}/2 + g_{1/2}(\text{dB})/2 \quad (6)$$

whence, by eq. (3),

$$G_2(\text{dBi}) = G_1(\text{dBi}) - g_{1/2}(\text{dB}). \quad (7)$$

To find the gain of the i^{th} antenna ($i = 3,4,5,\ldots$), it is sufficient to change the subscript 2 to i in eq. (7), and make the indicated evaluations of $g_{1/i}$.

Antenna Factor

From other theory[4], one has the far-field E-field antenna factor,

$$AFE^2 = 480\pi^2/\lambda^2 R_L G \quad (8)$$

from which, upon expressing λ in terms of f in MH_z, one has

$$G = 5.264\times 10^{-2} f^2/R_L AFE^2. \quad (9)$$

Substitute for G from eq. (9) into eqs (2) and (3), and rearrange to obtain

$$AFE_1 \times AFE_2 = 2.632 \times 10^{-2} f^2/R_L p^{1/2}\rho \quad (10)$$

and

$$AFE_2/AFE_1 = (g_{1/2})^{1/2}. \quad (11)$$

Solve eqs. (10) and (11) simultaneously for AFE_1 to find

$$AFE_1^2 = 2.632 \times 10^{-2} f^2/R_L p^{1/2}\rho\, (g_{1/2})^{1/2}. \quad (12)$$

In terms of decibels, i.e., taking 10 log of eq. (12) and remembering that AFE(dB) = 20 log AFE, one has

$$AFE_1(dB) = -15.797 + 20\log f - 10\log R_L - \Delta_{12}/2 - 10\log\rho - g_{1/2}(dB)2, \qquad (13)$$

and, from eq. (11), it follows that

$$AFE_2(dB) = AFE_1(dB) + g_{1/2}(dB). \qquad (14)$$

The expressions for AFE contain the same unknowns, which have to be determined by measurement, as do the expressions for G. An additional term involving the frequency appears. Since the antenna factors determined above are for far E-field to within the limitations imposed by the finite size of the antennas relative to the spacing and by other assumptions, the corresponding far-field magnetic field antenna factor AFH can be determined to the same accuracy by subtracting 51.527 dB (= 20 log 120π) from the AFE.

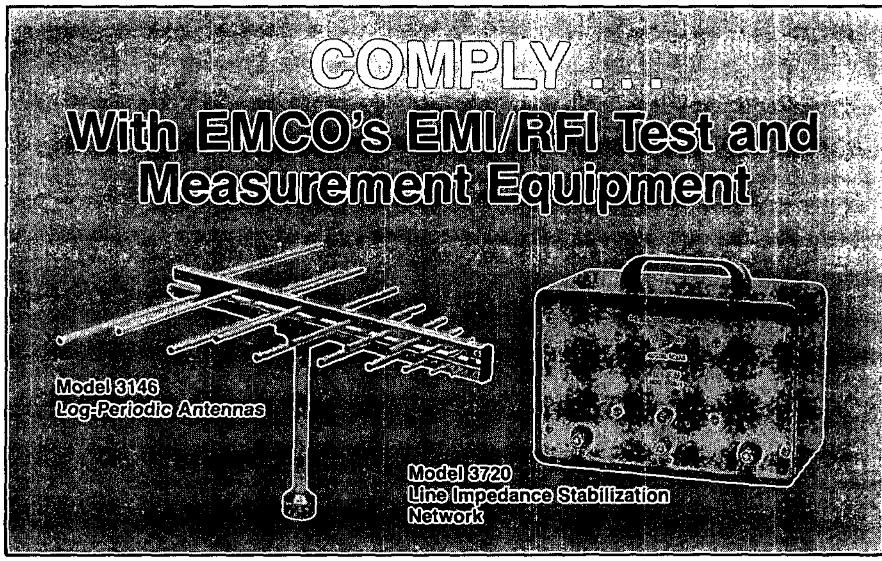
Case of Two Identical Antennas

When one has two identical passive, bilateral antennas, eq. (2) becomes

$$G^2 = 4p\rho^2, \qquad (15)$$

whereupon taking the common logarithms of both sides, and noting that 10 log p = Δ, the gain is obtained immediately as

$$G(dBi) = 3.010 + \Delta/2 + 10\log\rho. \qquad (16)$$



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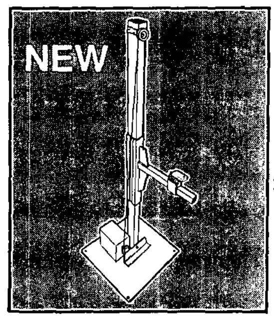

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Since the gains in this case are equal, $g_{1/2} = 1$, $g_{1/2}(dB) = 0$, and eqs. (12) and (13) reduce to

$$AFE^2 = 2.362 \times 10^{-2} f^2 / R_L p^{1/2} \rho \qquad (17)$$

and

$$AFE(dB) = -15.797 + 20 \log f - 10 \log R_L - \Delta/2 - 10 \log \rho \qquad (18)$$

The value of R_L for most instrumentation purposes is 50 ohms.

Example

Suppose it had been obtained for a given antenna at f = 420 MHz, that $\Delta_{12}/2 = 8.0$, R = 1 meter, $g_{1/2}(dB) = 2.0$. Then eqs. (6) and (13) yield for $R_L = 50$

$$G_1(dB) = 3.010 - 8.0 + 9.44 + 1.0 = 5.450 \text{ dBi} \qquad (19)$$

and

$$AFE_1(dB) = -32.787 + 52.46 + 8.0 - 9.44 - 1.0 = 17.23 \text{ dB}(m^{-1}). \qquad (20)$$

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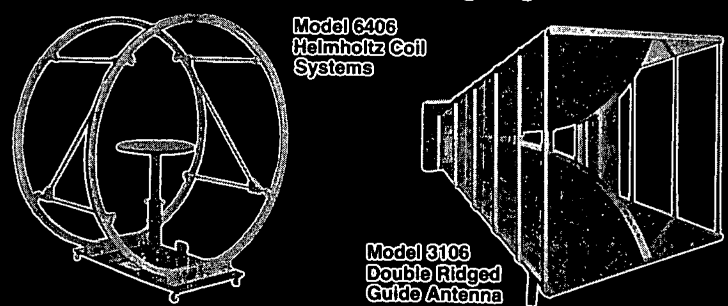

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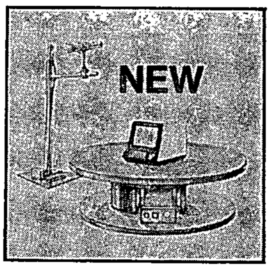

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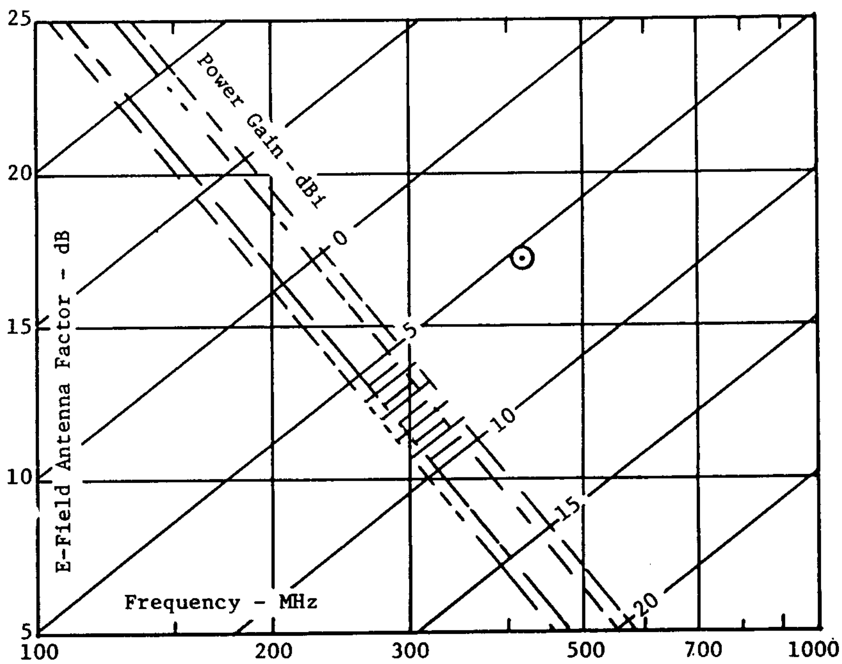

Figure 2. Typical AFE(dB)/G(dBi) Chart with Example.

Both points fall in the same place on an AFE/Gain chart (see Figure 2).

Procedure

1. Label numerically all the antennas to be measured or calibrated, and identify all recordings accordingly.

2. Use antennas Nos. 1 and 2 in a transmit-receive relationship to determine p at all frequencies of interest. In terms of decibels, p becomes Δ_{12}, which is the same as the overall insertion loss of the system, and is a negative number. A plotter is useful in this step by recording the response first by the radiation path and then by direct connection in feeder cables with a fixed safe number of decibels of attenuation introduced in place of the system. Record this fixed amount of attenuation, since this, plus the difference in response levels, amounts to Δ_{12}.

3. Determine $g_{1/2}$(dB) for antennas Nos. 1 and 2 by comparing the received signals in the same location with the auxiliary third antenna transmitting. Again, the plotter will facilitate this process by recording the response of each antenna on the same paper. The difference between responses in decibels is precisely g_{12}.

4. Determine 10 log ρ for all frequencies of interest. The chart shown in Figure 1 can be prepared for a prescribed spacing R, with frequency as the independent variable instead of radians. That is, for every frequency f (MHz) and fixed separation R, one has
$$r = (2\pi/300Rf = \text{constant} \times f).$$

5. Organize the data in columnar form against frequency to facilitate the calculation of gain and AFE from eqs. (6), (7), (13) and (14).

6. Present the results as desired, preferably as a plot on an appropriate AFE(dB)/G(dBi) chart as a function of frequency.

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1. H. Jasik, *Antenna Engineering Handbook*, McGraw-Hill, First Ed., 1961, pp 34-23 and 34-24.
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3. H. Jasik, *Op. Cit.*, p 3-2, 3-4.
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This article was written for ITEM by R. Wayne Masters, President, Antenna Research Associates, Inc., Beltsville, MD.